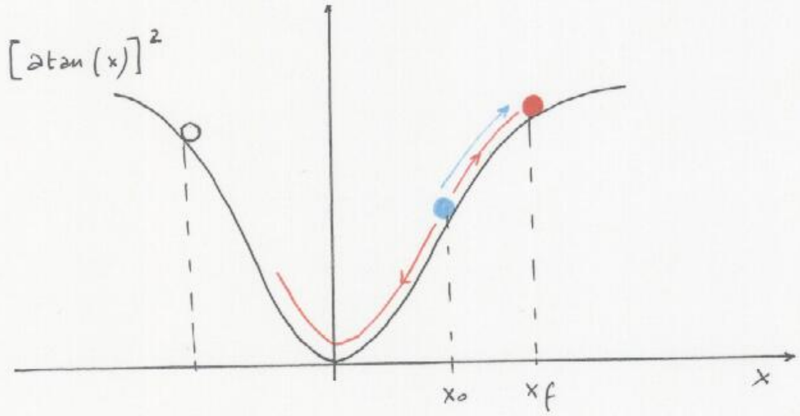
- Biorobotics : Lesson N° 9

- MATLAB

Mass in a potential Field :



to move the ball, from ● to ● we have to force with a const $\bar{F}$ if T isn't enough we can move toward ● directly ; but if T is enough we use the fact that the ball goes down and then, with more Energy goes to ●.

the problem is that there is a LOCAL MINIMA ; the problem was

$$\min_{p_0} \| x(t_f ; p_0) - x_f \|$$

$$\begin{cases} \dot{x} = \dfrac{\partial \mathcal{H}}{\partial p} \left(x(t), u^*\left(x(t), p(t)\right), p(t), t \right) \\ \\ \dot{p} = -\dfrac{\partial \mathcal{H}}{\partial x} \left(x(t), u^*\left(x(t), p(t)\right), p(t), t \right) \end{cases} \qquad \begin{array}{l} x(t_0) = x_0 , \\ x(t_f) = x_f \end{array}$$

⟶ this DYNAMICAL system has not an UNIC SOLUTION,
we can specify the p_0 initial

$C^* =$ cost optimal

Example: muscles seems to be springs

Stochastic Initialization of the Initial Conditions

the problem is:

$$\min_{u(\cdot)} \int_{t_0}^{t_f} \frac{1}{2} u^T(\tau) u(\tau) \, d\tau$$

$\downarrow$

Trajectory

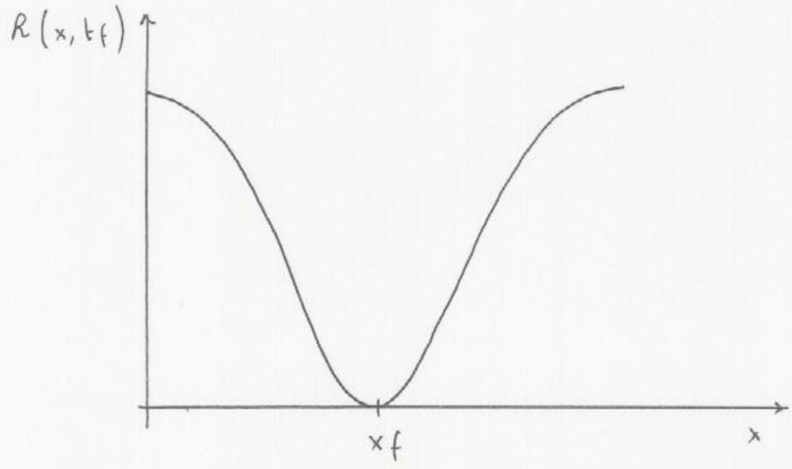
$$m \ddot{x} + b \dot{x} + \frac{2 \operatorname{atan}(x)}{1 + x^2} = u$$

$\longrightarrow$ solve this problem corresponds to solve the Dynamical system and the Force that moves the mass.

the initial conditions transform the trajectory in the search of two points

- The HAMILTON - JACOBI BELLMAN EQUATION

$$\min_{u(\cdot)} \int_{t_0}^{t_f} g\big(x(t), u(t), \tau\big) \, d\tau + h\big(x(t_f), t_f\big) \quad \text{s.t.} \quad \begin{cases} \dot{x} = f(x, u) \\ x(t_0) = x_0 \end{cases}$$



In a GENERAL FRAMEWORK, WE CAN THINK THAT AT EACH TIME INSTANT t THERE IS AN OPTIMAL TRAJECTORY TO BE FOLLOWED FROM THE CURRENT STATE TO THE TARGET STATE x_f.

AT TIME t, the COST to BE OPTIMIZED IS:

$$J\left(\bar{x}, t, \underset{t \le \tau \le t_f}{u(\tau)}\right) = \int_t^{t_f} g\left(x(\tau), u(\tau), \tau\right) d\tau + h\left(x(t_f), t_f\right)$$

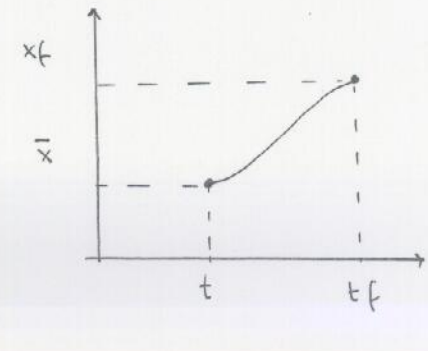
$$\text{s.t.} \quad \begin{cases} \dot{x} = f(x, u) \\ x(t) = \bar{x} \end{cases}$$

$J\left(x, t, \underset{t \le \tau \le t_f}{u(\tau)}\right)$ is called "COST TO GO"

the OPTIMAL CONTROL STRATEGY SHOULD MINIMIZE THE "COST TO GO" and THEREFORE WE CAN DEFINE the "OPTIMAL COST TO GO":

$$\boxed{J^*(\bar{x}, t)} = \min_{u(\cdot)} J\left(\bar{x}, t, \underset{t \le \tau \le t_f}{u(\tau)}\right)$$

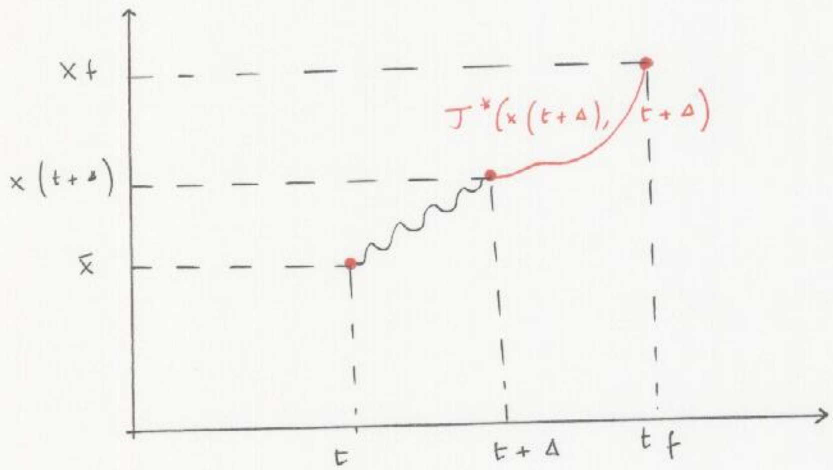
$$\text{s.t.} \quad \begin{cases} \dot{x} = f(x, u) \\ x(t) = \bar{x} \end{cases}$$



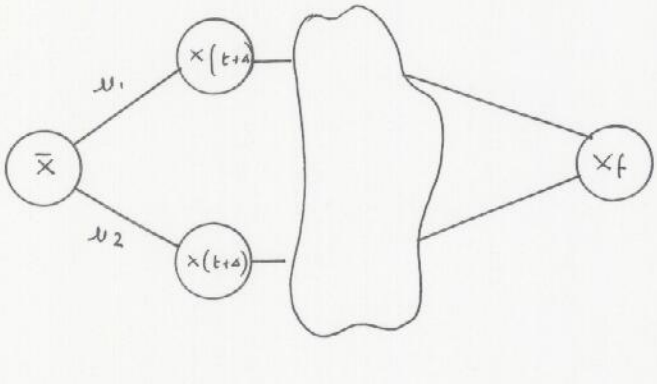
• CONDITIONS of $J^*(\bar{x}, t)$:

[1] $$J^*(\bar{x}, t) = \min_{u(\cdot)} \left\{ \int_t^{t+\Delta} g\big(x(\tau), u(\tau), \tau\big)\, d\tau + J^*\big(x(t+\Delta), t+\Delta\big) \right\}$$

where $x(t+\Delta)$ is the state reached at TIME $t+\Delta$ starting from the initial condition $\bar{x}$ at time t and applying the CONTROL STRATEGY $u(t)$, $t \in [t, t+\Delta]$



Es:



• $J^*(x, t_f) = h(x, t_f)$

It can be shown (taking the derivative with respect to time of [1]) that:

$$-\frac{\partial J^*}{\partial t}(t,x) = \min_u \left\{ \mathring{g}(x,u,t) + \left[\frac{\partial J^*}{\partial x}(x,t) \right]^T f(x,u) \right\}$$

HAMILTON JACOBI BELLMAN

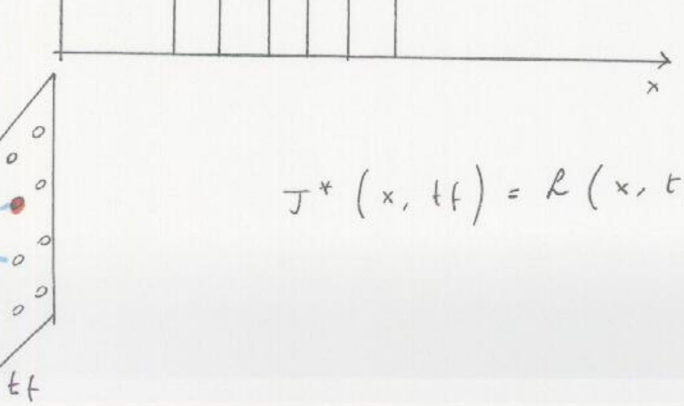
$$\text{s.t. } J^*(x,tf) = h(x,tf)$$

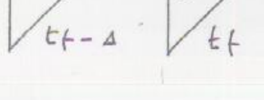
↳ if you solve this it's garantee that is an OPTIMAL SOLUTION

- DYNAMIC PROGRAMMING:

$x \in \{X_1 \dots X_N\}$ SET OF STATES

$u \in \{u_1 \dots u_N\}$ CONTROL INPUTS





$$J^*(x,tf) = h(x,tf)$$

$t \in [t_0, t_0 + \Delta \ \dots \ tf - \Delta, tf]$

(*)

- <u>MATLAB</u> :

Discretization of the space of variables

- Nv = 30 ;
- Nx = 50 ;
- Umin
- x Grid
- ifinal = fit to Grid
- V $\longrightarrow$ J^* % Function 'cost to go'
- computeV $\longrightarrow$ $J^*(x(t+\Delta), t+\Delta)$
- u star (i, T, k) $\longrightarrow$ it's yet written as a feedback, because you have the solution for each state
- images (x Grid, d dx Grid, V(:,:,1)) $\longrightarrow$ visualization of the cost
- colorbar

(*) then at time $tf - \Delta$, for a given value x of the current state, each discretized control strategy u_k applied in the interval $[tf - \Delta, tf]$ will bring the system to $x(t+\Delta)$ and the associated cost will be:

$$\int_t^{t+\Delta} g(x(\tau), u_k, \tau)\, d\tau + J^*(x(t+\Delta), tf)$$

or, if we make esplicit the fact that $x(\tau)$ depends on the choice of u_k we have $x(\tau, u_k)$ and therefore:

$$\int_t^{t+\Delta} g\left(x(\tau, u_k), u_k, \tau\right) + J^*\left(x(t+\Delta\, ; u_k), tf\right)$$

so that we have:

$$J^*(x, tf-\Delta) \simeq \min_{u_k \in \{u_i \dots u_k\}} \left\{ \int_{tf-\Delta}^{tf} g\left(x(\tau\, ; u_k), u_k, \tau\right) d\tau + \right. \\ \left. + J^*\left(x(t+\Delta\, ; u_k), tf\right) \right\}$$

where $x(\cdot\, ; u_k)$ is the solution of $\dot{x} = f(x, u_k)$

subject to $x(tf-\Delta) = x$