ESERCIZIO : MANIPOLATORE CASO PLANARE

invece che avere :

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \xrightarrow[red]{} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \longrightarrow \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \xrightarrow[red]{} \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

red = caso ridotto

la matrice di rotazione :

$$R = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow[red]{} \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix}$$

se θ varia nel tempo ,

$$\dot{R} R^T = S(\omega) \xrightarrow[red]{}$$

$$\omega = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \omega_z \end{bmatrix} \xrightarrow[red]{} \omega_z$$

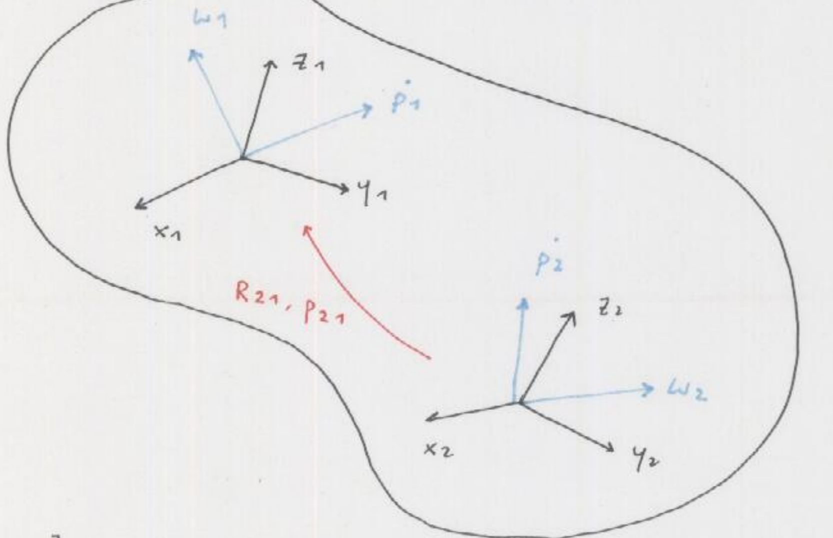
$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} \longrightarrow \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix} \xrightarrow[red]{} \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

$$\begin{bmatrix} \mu_x \\ \mu_y \\ \mu_z \end{bmatrix} \longrightarrow \begin{bmatrix} 0 \\ 0 \\ \mu_z \end{bmatrix} \xrightarrow[red]{} \mu_z$$

COPPIE sul piano $\perp$ alle For...

$$a \times b = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_y \\ 0 & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ a_x b_y - b_x a_y \end{bmatrix} \xrightarrow[red]{} a \times b = a_x b_y - a_y b_x = \det \begin{bmatrix} a_x & b_x \\ a_y & b_y \end{bmatrix}$$



$$\begin{bmatrix} \dot{p}_2^{\,1} \\ \omega_2^{\,2} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{21} & S(p_{21})R_2 \\ 0 & R_{21} \end{bmatrix}}_{Ad_{21} = Ad_{T_{21}}} \begin{bmatrix} \dot{p}_1^{\,1} \\ \omega_1^{\,1} \end{bmatrix}$$

ROTOTRASLAZIONE che VA DA 2 A 1

AGGIUNTA 21 ROTOTRASLAZ. RIGIDA

$$Ad_{21} = \begin{bmatrix} (R_{21})_{red} & \begin{bmatrix} p_{21}^y \\ -p_{21}^x \end{bmatrix} \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

COSA SUCCEDE alle FORZE-COPPIE:

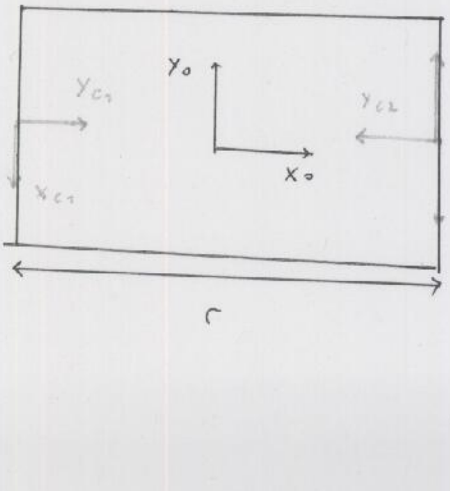
$$\begin{bmatrix} f_1^1 \\ \mu_1^1 \end{bmatrix} = \begin{bmatrix} R_{12} & 0 \\ S(p_{12})R_{12} & R_{12} \end{bmatrix} \begin{bmatrix} f_2^2 \\ \mu_2^2 \end{bmatrix}$$

$$\Big\downarrow \text{red}$$

$$\begin{bmatrix} (R_{12})_{red} & 0 \\ \left[-(p_{12})^y \;\; p_{12}^x\right](R_{12})_{red} & 1 \end{bmatrix}$$

$$\begin{bmatrix} (f_1^1)^x \\ (f_1^1)^y \\ (\mu_1^1)^z \end{bmatrix} = \begin{bmatrix} (R_{12})_{red} & 0 \\ \left[-p_{12}^y \; p_{12}^x\right](R_{12})_{red} & 1 \end{bmatrix} \begin{bmatrix} (f_2^2)^x \\ (f_2^2)^y \\ (\mu_2^2)^z \end{bmatrix}$$

- CALCOLA G e FC SAPENDO che i 2 CONTATTI SONO CON ATTRITO μ



N.B. LE TERNE DEVONO ESSERE DESTRORSE! (Z USCENTE).

$$B_{ci} = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} f_x \\ f_y \\ f_z \\ \mu_x \\ \mu_y \\ \mu_z \end{array} \qquad FC_i = \left\{ f \in \mathbb{R}^3 : \sqrt{f_1^2 + f_2^2} \leq \mu f_3 ,\ f_3 \geq 0 \right\}$$

$\longrightarrow$ red $(*)$

$$\begin{bmatrix} f_{ci}^{\,c_i} \\ \mu_{ci}^{\,c_i} \end{bmatrix} = B_{ci} f , \quad \forall f \in FC_i$$

Dato che siamo nel caso PLANARE, "buttiamo via" f_z, μ_x e μ_y ---

$$(*) \quad B_{ci} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{array}{l} f_x \\ f_y \\ \mu_z \end{array} ,$$

$$F_{ci} = \left\{ f \in \mathbb{R}^2 : |f_1| \leq \mu f_2 ,\ f_2 \geq 0 \right\}$$

✓ il ruolo dell'asse Z viene svolto dall'asse y

✓ portiamo dal s.rif. C_i al s.rif. 'O' :

$$\begin{bmatrix} f_o^{\,o} \\ \mu_o^{\,o} \end{bmatrix} = G f , \quad f \in FC$$

risultante dei 2 contatti

$$\begin{bmatrix} (f_{ci})^o \\ (\mu_{ci})^o \end{bmatrix} = \underbrace{\begin{bmatrix} R_{oci} & 0 \\ \left[-p_{oci}^{\,y} \quad p_{oci}^{\,x} \right] R_{oci} & 1 \end{bmatrix}}_{K_{oci}} \begin{bmatrix} f_{ci}^{\,c_i} \\ \mu_{ci}^{\,c_i} \end{bmatrix} =$$

$$= \begin{bmatrix} R_{oc_i} & 0 \\ \begin{bmatrix} -p_{oc_i}^{y} & p_{oc_i}^{x} \end{bmatrix} R_{oc_i} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} f_{c_i} \quad \forall f_{c_i} \in FC_i$$

$$\begin{bmatrix} f_o^o \\ \mu_o^o \end{bmatrix} = \begin{bmatrix} (f_{c_1})^o \\ (\mu_{c_1})^o \end{bmatrix} + \begin{bmatrix} (f_{c_2})^o \\ (\mu_{c_2})^o \end{bmatrix}$$

$$R_{oc_1} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

MATRICE DI ROT. DA C1 A O

$$p_{oc_1} = \begin{bmatrix} -r \\ 0 \end{bmatrix}$$

$$R_{oc_2} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$p_{oc_2} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

$$K_{oc_i} = \begin{bmatrix} 0 & -r \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} r & 0 \end{bmatrix}$$

$$H_{oc2} = \begin{bmatrix} 0 & r \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} r & 0 \end{bmatrix}$$

NOTA: L'ES D'ESAME SARÀ SIMILE A QUESTO CON PUNTI DI CONTATTO DIVERSI!
(TIPO UNO SOPRA E UNO SOTTO)

$$\begin{bmatrix} f^\circ \\ \mu_o^\circ \end{bmatrix} = \begin{bmatrix} (f_{c1})^\circ \\ (\mu_{c1})^\circ \end{bmatrix} + \begin{bmatrix} (f_{c2})^\circ \\ (\mu_{c2})^\circ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ r & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} f_{c1} +$$

$$+ \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ r & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} f_{c2} = (*)$$

$$f_{c1} \in FC_1 = \left\{ f \in \mathbb{R}^2 : \| f_1 \| \leq \mu f_2,\ f_2 \geq 0 \right\} = FC_2$$

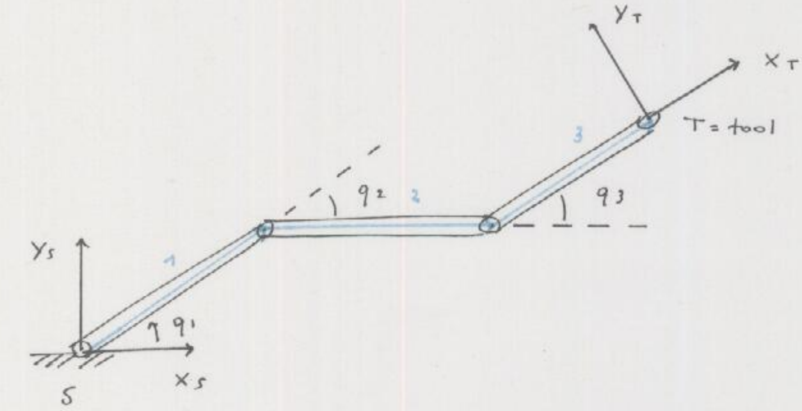
$$f_{c2} \in FC_2$$

$$\begin{bmatrix} f_o^\circ \\ \mu_o^\circ \end{bmatrix} = G f \qquad f \in FC$$

$$(*) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ r & 0 \end{bmatrix} f_{c1} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ r & 0 \end{bmatrix} f_{c2}$$

$$= \underbrace{\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ r & 0 & r & 0 \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} f_{c1} \\ f_{c2} \end{bmatrix}}_{f} \qquad f \in \underbrace{FC_1 \times FC_2}_{FC}$$

ESERCIZIO N° 2: MANIPOLATORE PLANARE A 3 G.D.L.



si calcoli $J^S_{ST}(q)$ ricordando:

$$J^S_{ST}(q) = Ad_{ST} \begin{bmatrix} R_{ST}^T & 0 \\ 0 & R_{ST}^T \end{bmatrix} J_{ST}(q)$$

DOVE $J_{ST}(q)$ e' lo Jacobiano GEOMETRICO DEL SISTEMA

SCRIVO LA TRASFORMAZIONE da S a T

$$p_{ST} = \underbrace{\begin{bmatrix} l_1 \, c\, q_1 \\ l_1 \, s\, q_1 \end{bmatrix}}_{\bullet 1} + \underbrace{\begin{bmatrix} l_2 \, c\, q_1+q_2 \\ l_2 \, s\, q_1+q_2 \end{bmatrix}}_{\bullet 2} + \underbrace{\begin{bmatrix} l_3 \, c\, q_1+q_2+q_3 \\ l_3 \, s\, q_1+q_2+q_3 \end{bmatrix}}_{\bullet 3}$$

$$R_{ST} = \begin{bmatrix} c\, q_1+q_2+q_3 & -s\, q_1+q_2+q_3 \\ s\, q_1+q_2+q_3 & c\, q_1+q_2+q_3 \end{bmatrix}$$

$$J_{ST}(q)\dot{q} = \begin{bmatrix} \dot{p}_{ST} \\ \omega_{ST} \end{bmatrix}$$

(def Jacobiano geometrico

$$R_{ST} = \begin{bmatrix} c_{q_1+q_2+q_3} & -s_{q_1+q_2+q_3} & 0 \\ s_{q_1+q_2+q_3} & c_{q_1+q_2+q_3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{R}_{ST} R_{ST}^T = S(\omega) \longrightarrow \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 + \dot{q}_2 + \dot{q}_3 \end{bmatrix}$$

$$J_{ST}(q)\dot{q} = \begin{bmatrix} \dot{p}_{ST} \\ \omega_{ST} \end{bmatrix} \longrightarrow \dot{q}_1 + \dot{q}_2 + \dot{q}_3$$

$$\dot{p}_{ST} = \begin{bmatrix} \left(-l_1 s_{q_1} - l_2 s_{q_1+q_2} - l_3 s_{q_1+q_2+q_3}\right)\dot{q}_1 - \left(l_2 s_{q_1+q_2} + l_3 s_{q_1+q_2+q_3}\right)\dot{q}_2 + \dots \\ \dots \left(-l_3 s_{q_1+q_2+q_3}\right)\dot{q}_3 \\ \left(l_1 c_{q_1} + l_2 c_{q_1+q_2} + l_3 c_{q_1+q_2+q_3}\right)\dot{q}_1 + \left(l_2 c_{q_1+q_2} + l_3 c_{q_1+q_2+q_3}\right)\dot{q}_2 + \dots \\ \dots + \left(l_3 c_{q_1+q_2+q_3}\right)\dot{q}_3 \end{bmatrix} \begin{matrix} (*) \\ \\ (**) \\ \\ \end{matrix}$$

$$\begin{bmatrix} \dot{p}_{ST} \\ \omega_{ST} \end{bmatrix} = \begin{bmatrix} (*) \\ (**) \\ \dot{q}_1 + \dot{q}_2 + \dot{q}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -l_1 s_{q_1} - l_2 s_{q_1 q_2} - l_3 s_{q_1+q_2+q_3} & \dots \\ l_1 c_{q_1} + l_2 c_{q_1+q_2} + l_3 c_{q_1+q_2+q_3} & \dots \\ 1 & \dots \end{bmatrix}}_{\dots}$$

$$\underbrace{\begin{bmatrix} \dots & -l_2 s_{q_1+q_2} - l_3 s_{q_1+q_2+q_3} & -l_3 s_{q_1+q_2+q_3} \\ \dots & l_2 c_{q_1+q_2} + l_3 c_{q_1+q_2} & l_3 c_{q_1+q_2+q_3} \\ \dots & 1 & 1 \end{bmatrix}}_{J_{ST}(q)} \underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}}_{\dot{q}}$$

NOTA: SPESSO SOSTITUISCE UN VINCOLO ROTAZIONALE CON VINCOLO PRISMATICO
(SEMPLIFICA LE COSE! HO UNA $\dot{q}$ IN MENO!)

$$Ad_{ST} = \begin{bmatrix} R_{ST} & S(p_{ST})\,R_{ST} \\ 0 & R_{ST} \end{bmatrix}$$

$$Ad_{ST} \cdot \begin{bmatrix} R_{ST}^T & 0 \\ 0 & R_{ST}^T \end{bmatrix} = \begin{bmatrix} R_{ST} & S(p_{ST})\,R_{ST} \\ 0 & R_{ST} \end{bmatrix} \begin{bmatrix} R_{ST}^T & 0 \\ 0 & R_{ST}^T \end{bmatrix} =$$

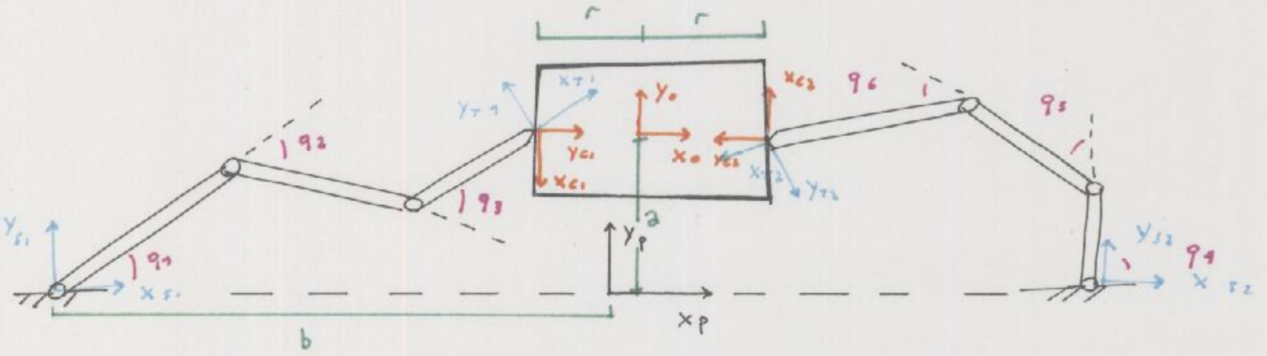
$$= \begin{bmatrix} I & S(p_{ST}) \\ 0 & I \end{bmatrix} \longrightarrow \begin{bmatrix} I_{2\times 2} & \begin{bmatrix} p_{ST}^y \\ -p_{ST}^x \end{bmatrix} \\ 0 & 1 \end{bmatrix}$$

$$J_{ST}^s(q) = \begin{bmatrix} 1 & 0 & p_{ST}^y \\ 0 & 1 & -p_{ST}^x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ 1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cancel{J_{11} + p_{ST}^y} & J_{12} + p_{ST}^y & J_{13} + p_{ST}^y \\ \cancel{J_{21} - p_{ST}^x} & J_{22}\ p_{ST}^x & J_{23} - p_{ST}^x \\ 1 & 1 & 1 \end{bmatrix}$$

$$J_{ST}^s(q) = \begin{bmatrix} 0 & l_1 s q_1 & l_1 s q_1 + l_2 s q_1{+}q_2 \\ 0 & -l_1 c q_1 & -l_1 c q_1 - l_2 c q_1{+}q_2 \\ 1 & 1 & 1 \end{bmatrix}$$

QUESITO FINALE ESERCIZIO 2:



- • SISTEMA SOLIDALE ALL'OGGETTO ;
- • SISTEMA SOLIDALE AL DITO ;

- f → finger
- b → body

SI CALCOLINO I VINCOLI CINEMATICI TRA LE COORDINATE DI GIUNTI

$$q_{f_1} = [\, q_1, q_2, q_3 \,]^T$$

$$q_{f_2} = [\, q_4, q_5, q_6 \,]$$

E LA velocità dell'OGGETTO V_{po}^{b}, ovvero l'equazione:

$$J_h(q)\,\dot{q} = G^T V_{po}^{b} \quad , \quad \dot{q} = \begin{bmatrix} \dot{q}_{f_1} \\ \dot{q}_{f_2} \end{bmatrix}$$

$\downarrow$

$J_R(q, x_o)$ = non dipende dalla pos. dell'oggetto rispetto al palmo

Ci manca da calcolare la MATRICE : $J_R(q)$

$$J_R(q) = \begin{bmatrix} B_{c_1}^T \, Ad_{s_1 c_1}^{-1} \, J_{s_1 T_1}^{s}(q) & 0 \\ 0 & B_{c_2}^T \, Ad_{s_2 c_2}^{-1} \, J_{s_2 T_2}^{s}(q_{f_2}) \end{bmatrix}$$

$$Ad_{s_1 c_1}^{-1} = Ad_{c_1 s_1} = \begin{bmatrix} R_{c_1 s_1} & \begin{bmatrix} p_{c_1 s_1}^{y} \\ -p_{c_1 s_1}^{x} \end{bmatrix} \\ 0 & 1 \end{bmatrix}$$

$$R_{c_1 s_1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

ROTOTRASLAZ. che porta punti descritti in S_1 in C_1, faccio il "giro": $T_1 \rightarrow S_1 \rightarrow C_1$

$$p_{c_1 s_1} = \begin{bmatrix} a \\ r-b \end{bmatrix} \implies Ad_{c_1 s_1} = \begin{bmatrix} 0 & -1 & r-b \\ 1 & 0 & -a \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{c_2 s_2} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$p_{c_2 s_2} = \begin{bmatrix} -a \\ r-b \end{bmatrix}$$

$$Ad_{c_2 s_2} = \begin{bmatrix} 0 & 1 & r-b \\ -1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_{c_1}^{T} Ad_{s_1 c_1}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & r-b \\ 1 & 0 & -a \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & r-b \\ 1 & 0 & -a \end{bmatrix}$$

$$B_{c_2}^{T} Ad_{s_2 c_2}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & r-b \\ -1 & 0 & a \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & r-b \\ -1 & 0 & a \end{bmatrix}$$

$$B_{c_1} \, Ad^{-1}_{s_1 c_1} \, J^{s}_{s_1 T_1}(q_{f_1}) = \begin{bmatrix} 0 & -1 & r-b \\ 1 & 0 & -a \end{bmatrix} \begin{bmatrix} 0 & \ell_1 s q_1 & \ell_1 s q_1 + \ell_2 s q_1{+}q_2 \\ 0 & -\ell_1 c q_1 & -\ell_1 c q_1 - \ell_2 c q_1{+}q_2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r-b & \ell_1 c q_1 + (r-b) & \ell_1 c q_1 + \ell_2 c q_1{+}q_2 + r - b \\ -a & \ell_1 s q_1 - a & \ell_1 s q_1 + \ell_2 s q_1{+}q_2 - a \end{bmatrix}$$

$$B_{c_2}^{T} \, Ad^{-1}_{s_2 c_2} \, J_{s_2 T_2}(q_{f_2}) = \begin{bmatrix} 0 & 1 & r-b \\ -1 & 0 & a \end{bmatrix} \begin{bmatrix} 0 & \ell_1 s q_4 & \ell_1 s q_4 + \ell_2 s q_4{+}q_5 \\ 0 & -\ell_1 c q_4 & -\ell_1 c q_4 - \ell_2 c q_4{+}q_5 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} r-b & -\ell_1 c q_4 + (r-b) & \overset{(*)}{-\ell_1 c q_4 - \ell_2 c q_4{+}q_5 + (r-b)} \\ a & \ell_1 s q_4 + a & \overset{(**)}{\ell_1 s q_4 + \ell_2 s q_4{+}q_5 + a} \end{bmatrix}$$

$$J_R(q) = \begin{bmatrix} r-b & \ell_1 c q_1 + r-b & \ell_1 q_1 + \ell_2 c q_1{+}q_2 + r-b & 0 & 0 & 0 \\ -a & -\ell_1 s q_1 - a & -\ell_1 s q_1 - \ell_2 s q_1{+}q_2 - a & 0 & 0 & 0 \\ 0 & 0 & 0 & r-b & -\ell_1 c q_4 + r-b & (*) \\ 0 & 0 & 0 & a & \ell_1 s q_4 + a & (**) \end{bmatrix}$$

$$\cdot \begin{bmatrix} \dot q_1 \\ \dot q_2 \\ \dot q_3 \\ \dot q_4 \\ \dot q_5 \\ \dot q_6 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 & -1 & r \\ 1 & 0 & 0 \\ 0 & 1 & r \\ -1 & 0 & 0 \end{bmatrix}}_{G^T} \begin{bmatrix} \dot x_0 \\ \dot y_0 \\ \dot \theta_0 \end{bmatrix}$$