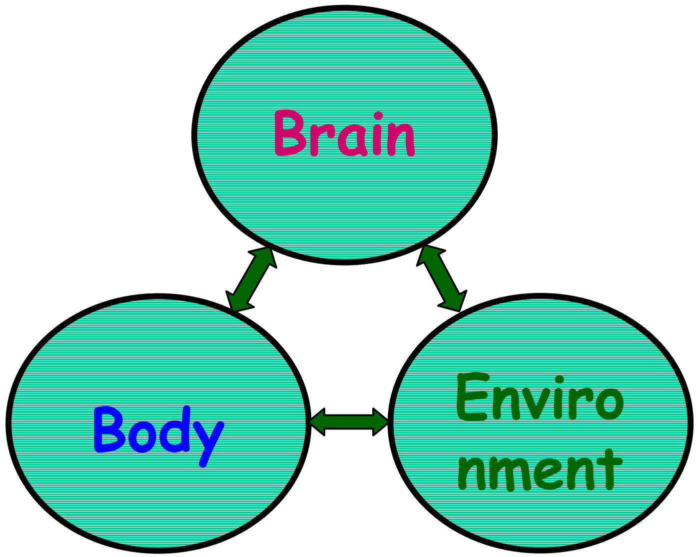
ISTITUTO ITALIANO
DI TECNOLOGIA

RBCS dept

motor learning & rehab lab

The fil rouge of the lab

- Skilled actions can be represented by means of multiple, interacting force fields
- Motor learning can be viewed as the job of optimally shaping a force field for a given task
- Recovering a motor function, in neuromotor rehabilitation, is equivalent to learning a motor task
- Motor cognition is shaped by the dynamic/computational interaction with the body and the environment



The underlying theoretical framework

- Schema theory (motor learning)
- Equilibrium point hypothesis (action representation & execution)

EPH is a general language for describing motor control in humans and humanoid robots.

But it is a language with many dialects.

Its power comes from the ability to solve the "degrees of freedom problem" as formulated by Nikolai Bernstein

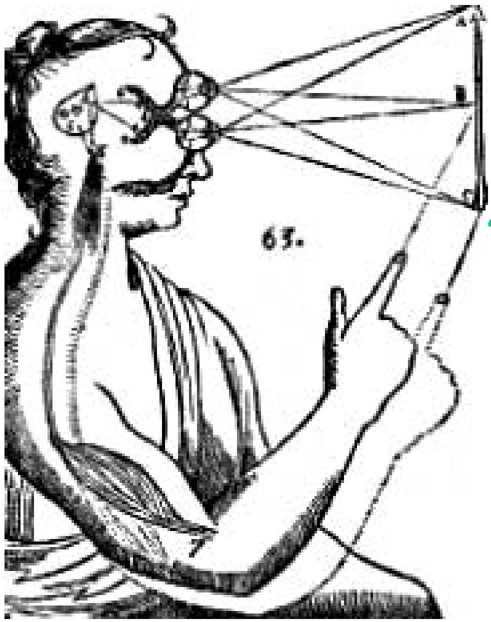
Equilibrium ⇔ Energy functions ⇔ Force fields

- Physical force fields
  - muscle properties
- Virtual force fields
  - attractor dynamics of cortical maps / motor imagery
- Physical force fields
  - haptic robots / haptic interaction in physiotherapy

From **EPH** (Equilibrium Point Hypothesis) to **PMP** (Passive Motion Paradigm):

a general computational model of synergy formation

Inverse kinematics



Task space: $x(t) \in R^n$

Joint space: $q(t) \in R^m$

$$x(t) = f(q)$$

$$\dot{x} = J(q)\,\dot{q} \qquad J(q) = \frac{\partial x}{\partial q}$$

Algebraic solution:

Non-redundant case (n=m)

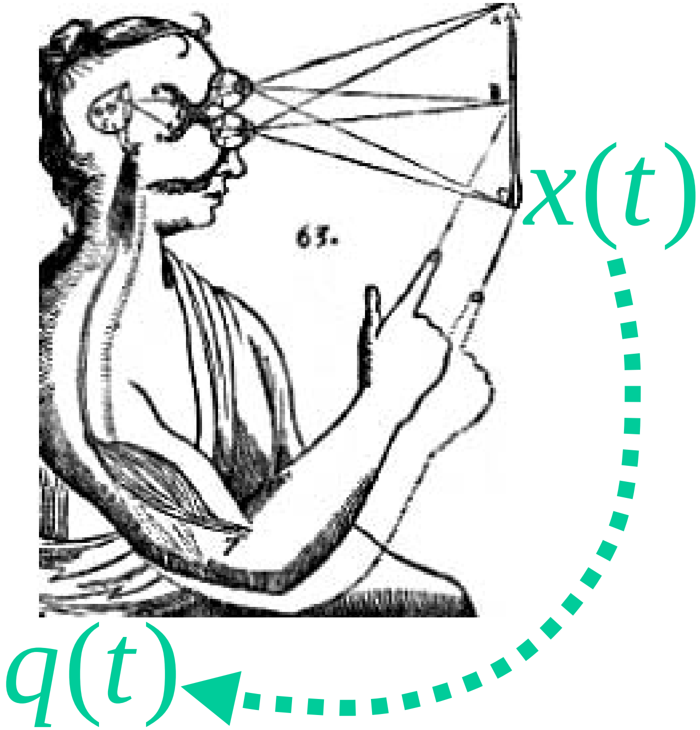
$$\dot{q} = J(q)^{-1}\,\dot{x}$$

Redundant case (n<m)

$$\dot{q} = J(q)^T [J(q) J(q)^T]^{-1}\,\dot{x}$$

Moore-Penrose pseudo-inverse matrix

Inverse kinematics (Moore-Penrose solution)



$$\dot{q} = J(q)^T [J(q)J(q)^T]^{-1} \dot{x}$$

It minimizes $|\dot{q}|$ given $\dot{x}$

Kinematic singularities:

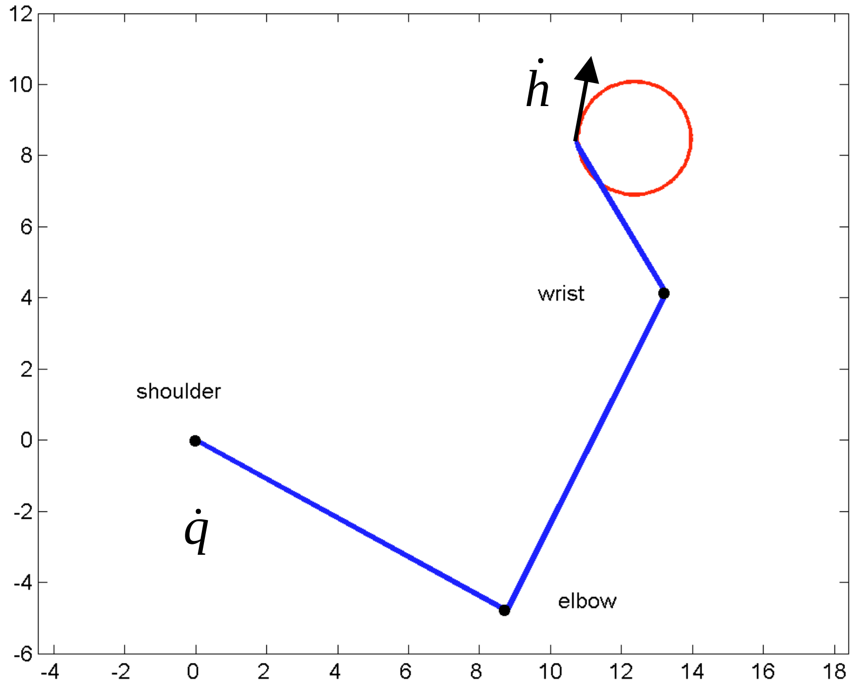
$$\det[J(q)J(q)^T] = 0$$

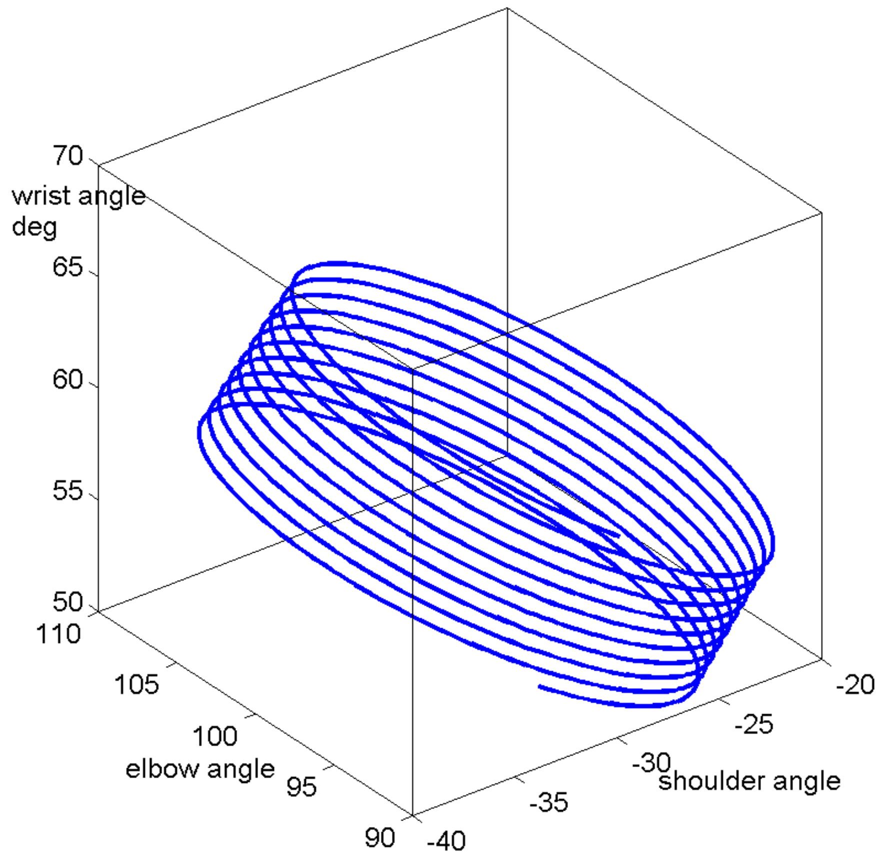
Null space:

$$0 = J(q)\dot{q}$$

Inverse kinematics via the Moore-Penrose matrix

$$\dot{q} = J(q)^T \left(J(q)J(q)^T\right)^{-1} \dot{h} \;\;\Rightarrow\;\; q(t) = \int J(q)^T \left(J(q)J(q)^T\right)^{-1} \dot{h}\, dt$$





the solution is not integrable

Inverse statics

Problem

Given an external force F compute the torque vector τ which can equilibrate F.

Solution

$$\tau = J(q)^T F$$

The solution is found by means of the principle of virtual works:

- Virtual movement $\delta q \to \delta h$
- Equality of virtual works: $\delta h^T F = \delta q^T \tau$, but $\delta h = J \delta q$
- Then $\delta q^T J^T F = \delta q^T \tau$ from which the solution

The same approach can be used for an external force, applied to the end-effector, and for the force of gravity, applied to the COM of each link.

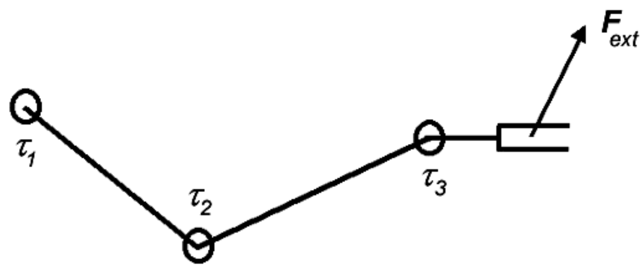
Principle of duality between joint space and task space

Task space $\qquad \dot{h} = J(q)\dot{q} \qquad$ Joint space

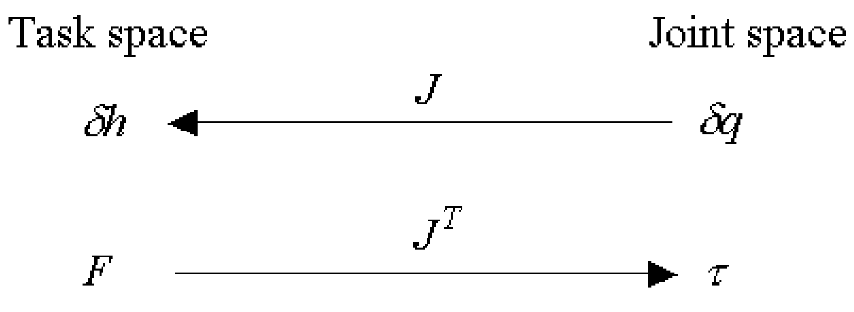
$$\delta h \xleftarrow{\quad J \quad} \delta q$$

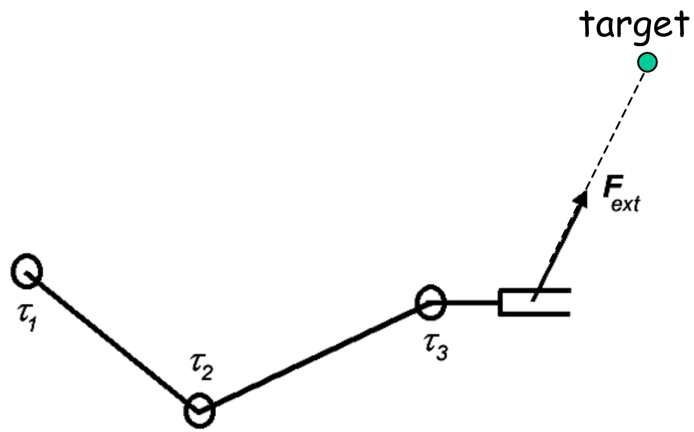
$$F \xrightarrow{\quad J^T \quad} \tau$$

$$\tau = J(q)^T F$$



Incremental kinematics + Inverse statics $\Rightarrow$ Passive Motion Paradigm





Stiffness matrix

Force field

Admittance matrix

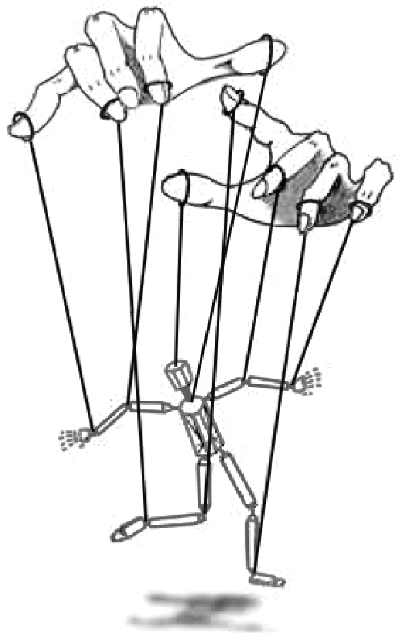
$$F_{ext} = K\left(x_{\text{target}} - x_{\text{hand}}\right) \Rightarrow \tau = J^T(q) \Rightarrow dq = A\tau \Rightarrow dh = J(q)dq$$

Mussa Ivaldi, F. A., Morasso, P., Zaccaria, R. (1988) Kinematic Networks. A Distributed Model for Representing and Regularizing Motor Redundancy. Biological Cybernetics, 60, 1-16.

PMP (passive motion paradigm)

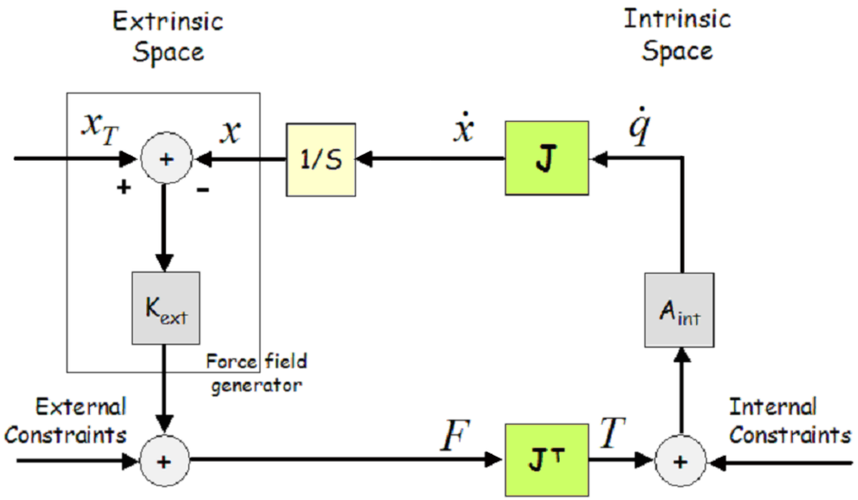
The coordination of a highly redundant set of DoF's is similar to the (passive) motion of an internal Body Schema under the action of a task-related virtual force field, combined with additional constraint-related force fields.

Motor imagery

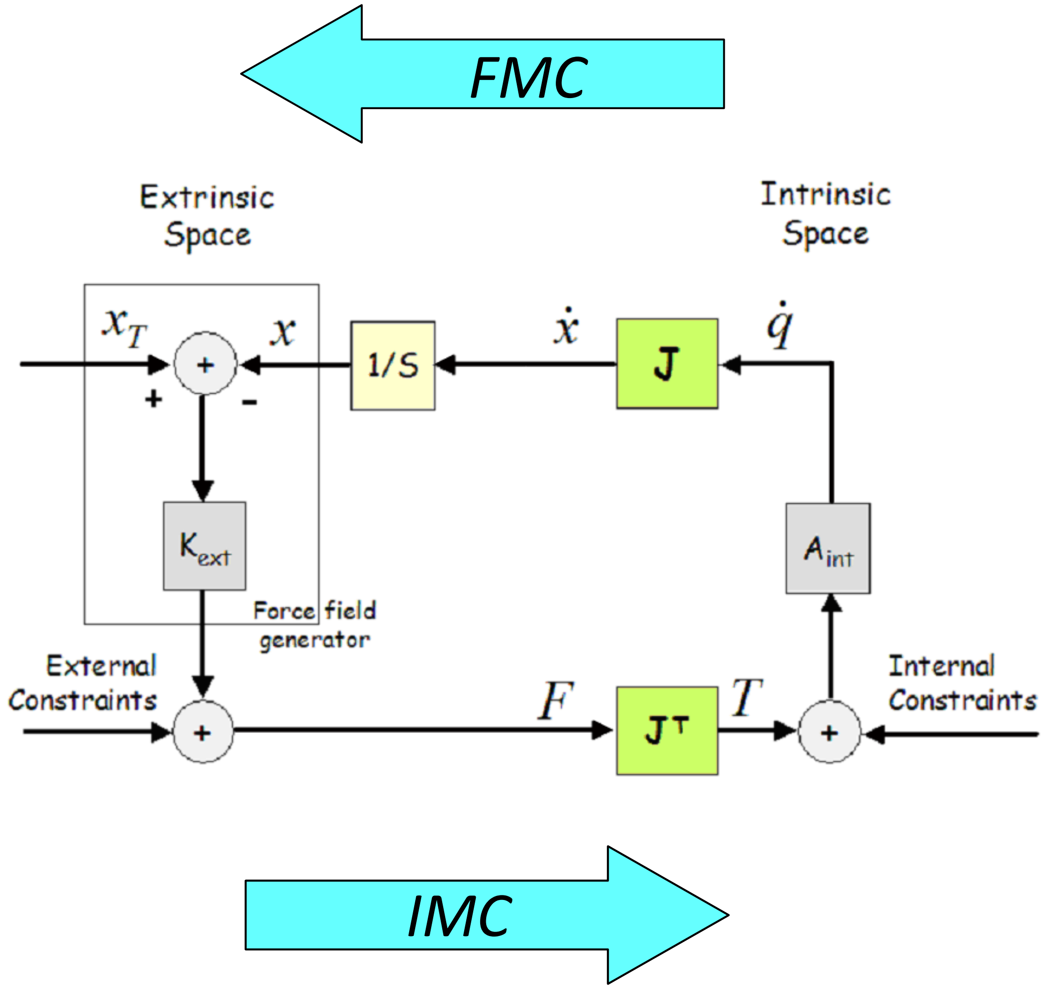


Force fields

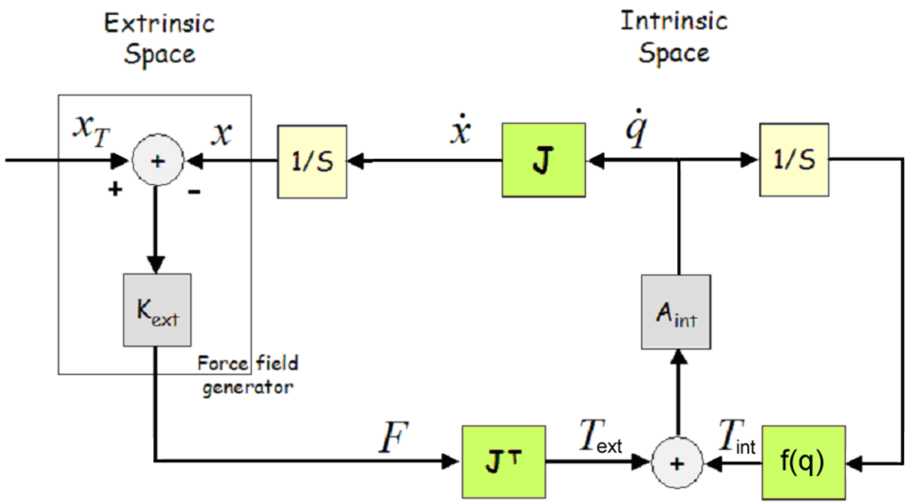
Body schema

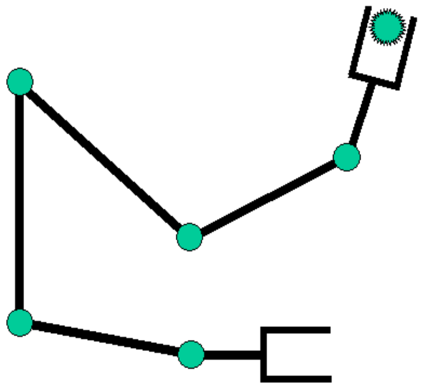


Mussa Ivaldi F A, Morasso P, Zaccaria R (1988) Kinematic Networks. A Distributed Model for Representing and Regularizing Motor Redundancy. Biol Cybern 60:1-16.

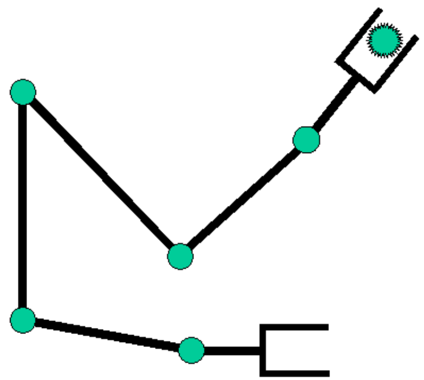


- It solves implicitly the "degrees of freedom problem"
- There is no fixed hierarchy between the extrinsic and intrinsic spaces.
- Gracefuld degradation if the target is outside the workspace.
- It includes the concurrent action of a FMC (Forward Motor Controller) and IMC (Inverse Motor Controller)

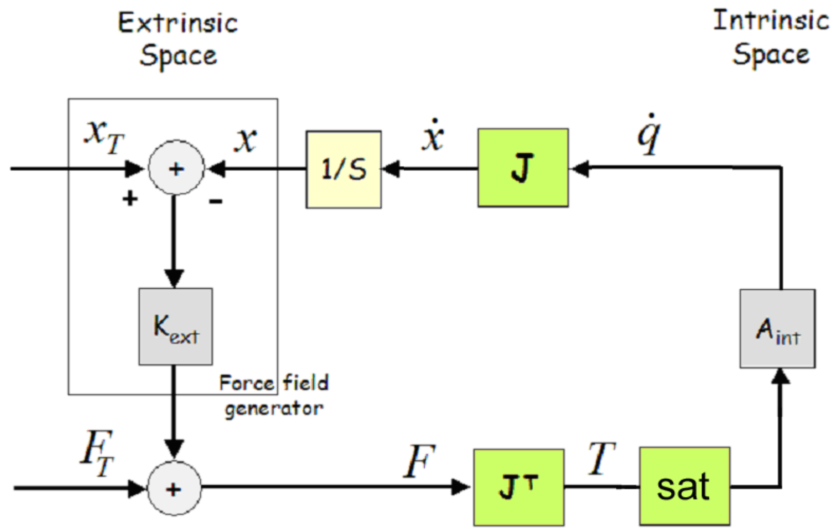




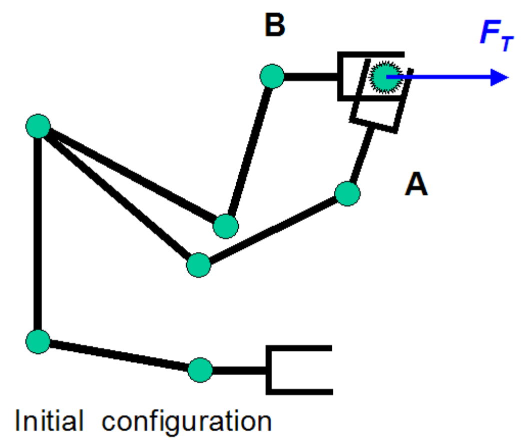
Initial configuration



Initial configuration

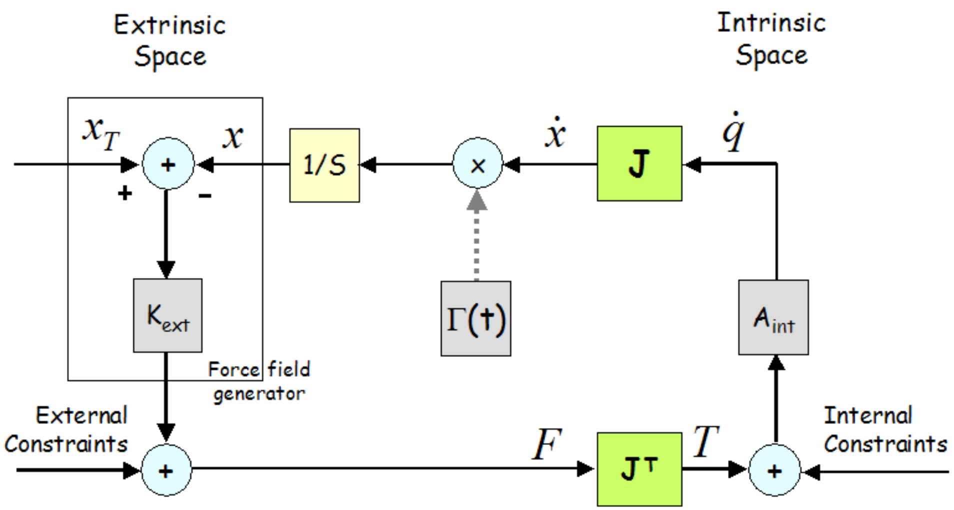


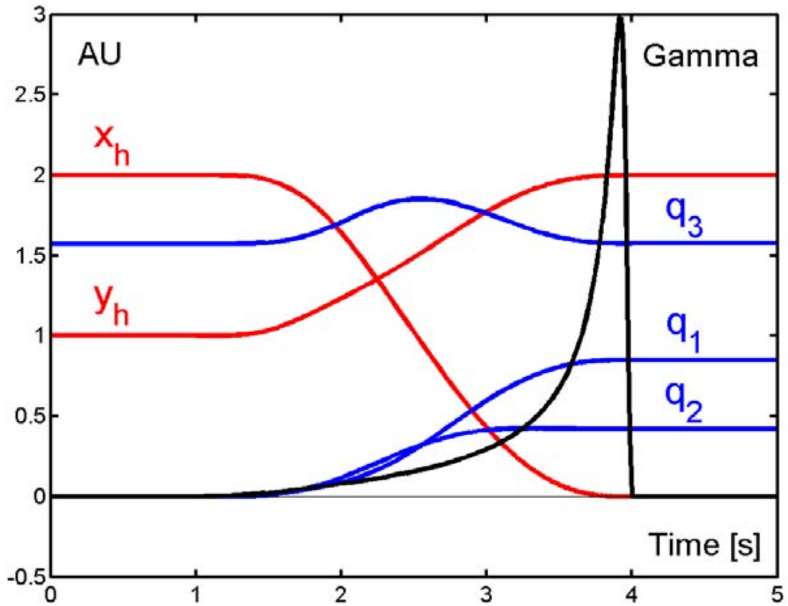
Extrinsic Space
Intrinsic Space
x_T
x
$\dot{x}$
$\dot{q}$
+
-
1/S
J
K_{ext}
A_{int}
Force field generator
F_T
F
T
J^T
sat



B
F_T
A
Initial configuration

Terminal attractor dynamics: indirect control of timing





$\Gamma(t)$: time base generator

$$
\begin{cases}
\Gamma(t) = \dot{\xi} / (1-\xi) \\
\xi(t) = 6 \cdot (t/T)^5 - 15(t/T)^4 + 10(t/T)^3
\end{cases}
$$

Zak, M. (1988) Terminal attractors for addressable memory in neural networks. Phys. Lett. A, 133, 218–222.

Bullock, D., Grossberg, S. (1988) Neural dynamics of planned arm movements: emergent invariants and speed-accuracy properties. Psychol Rev, 95, 49–90

Tsuji, T., Morasso, P., Shigehashi, K., Kaneko, M. (1995) Motion Planning for Manipulators using Artificial Potential Field Approach that can Adjust Convergence Time of Generated Arm Trajectory. Journal Robotics Society of Japan, 13, 285-290.

Terminal attractor dynamics: indirect control of timing

$$\frac{dx}{dt} = -Kx \longrightarrow x = x_o e^{-Kt}$$

The origin x=0 is an asymptotic attractor, reached in ∞ time

$$\frac{dx}{dt} = -\Gamma(t)\,Kx \longrightarrow x = x_o(1-\xi)^K$$

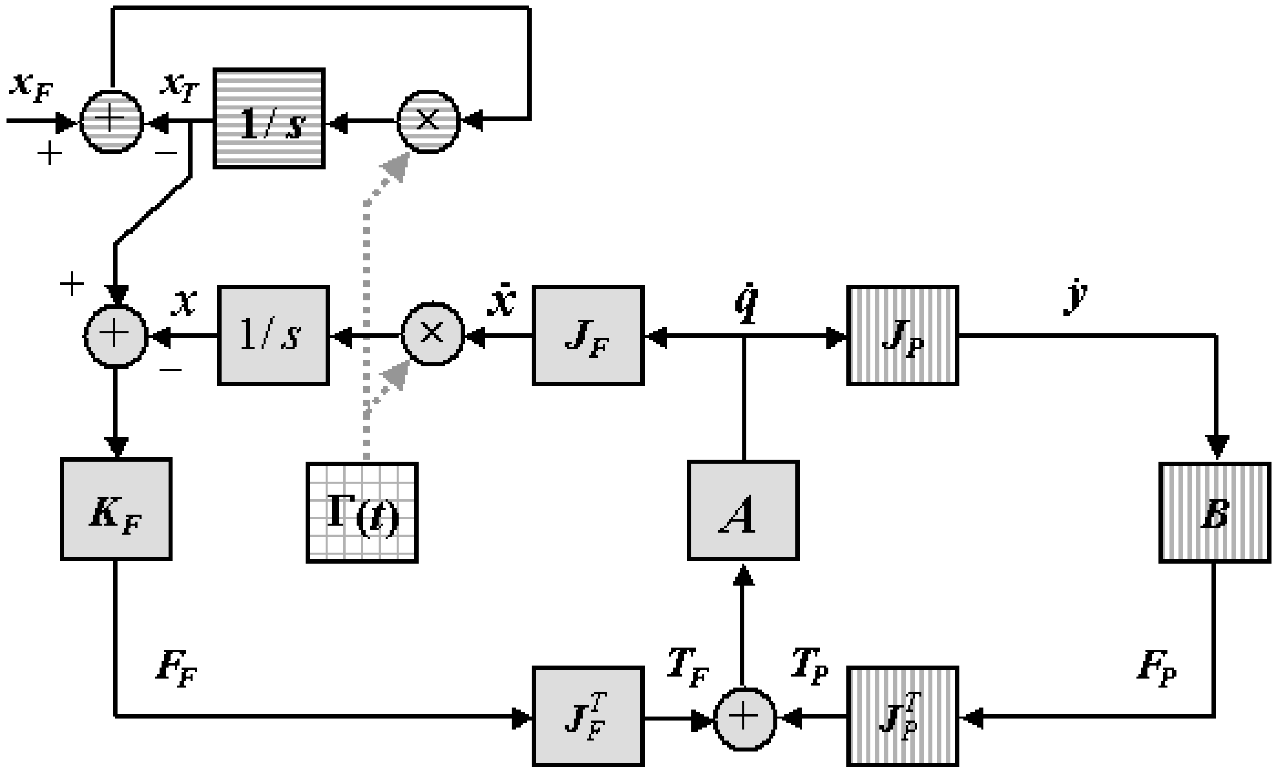
The origin x=0 is a "terminal" attractor, reached in finite time

Demonstration

$$\begin{cases} \Gamma(t) = \dot{\xi} / (1-\xi) \\ \xi(t) = 6 \cdot (t/\tau)^5 - 15(t/\tau)^4 + 10(t/\tau)^3 \end{cases}$$

$$\frac{dx}{dt} = -\Gamma(t)\,Kx \longrightarrow \frac{dx}{dt} = -\frac{d\xi/dt}{1-\xi}Kx \longrightarrow \frac{dx}{d\xi} = -\frac{Kx}{1-\xi}$$

Synergy formation in WBR (Whole Body Reaching)

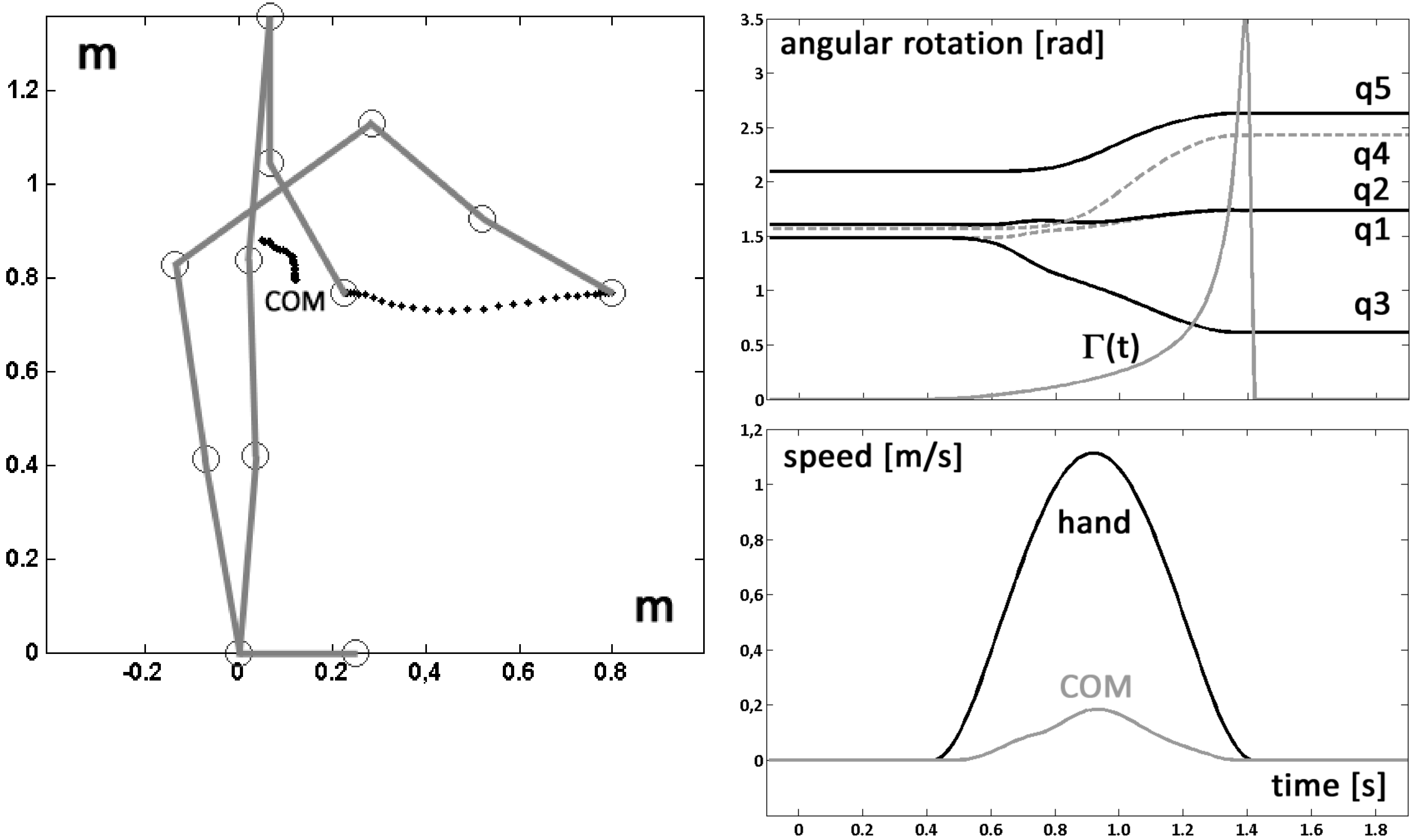


Focal part of the synergy

Postural part of the synergy

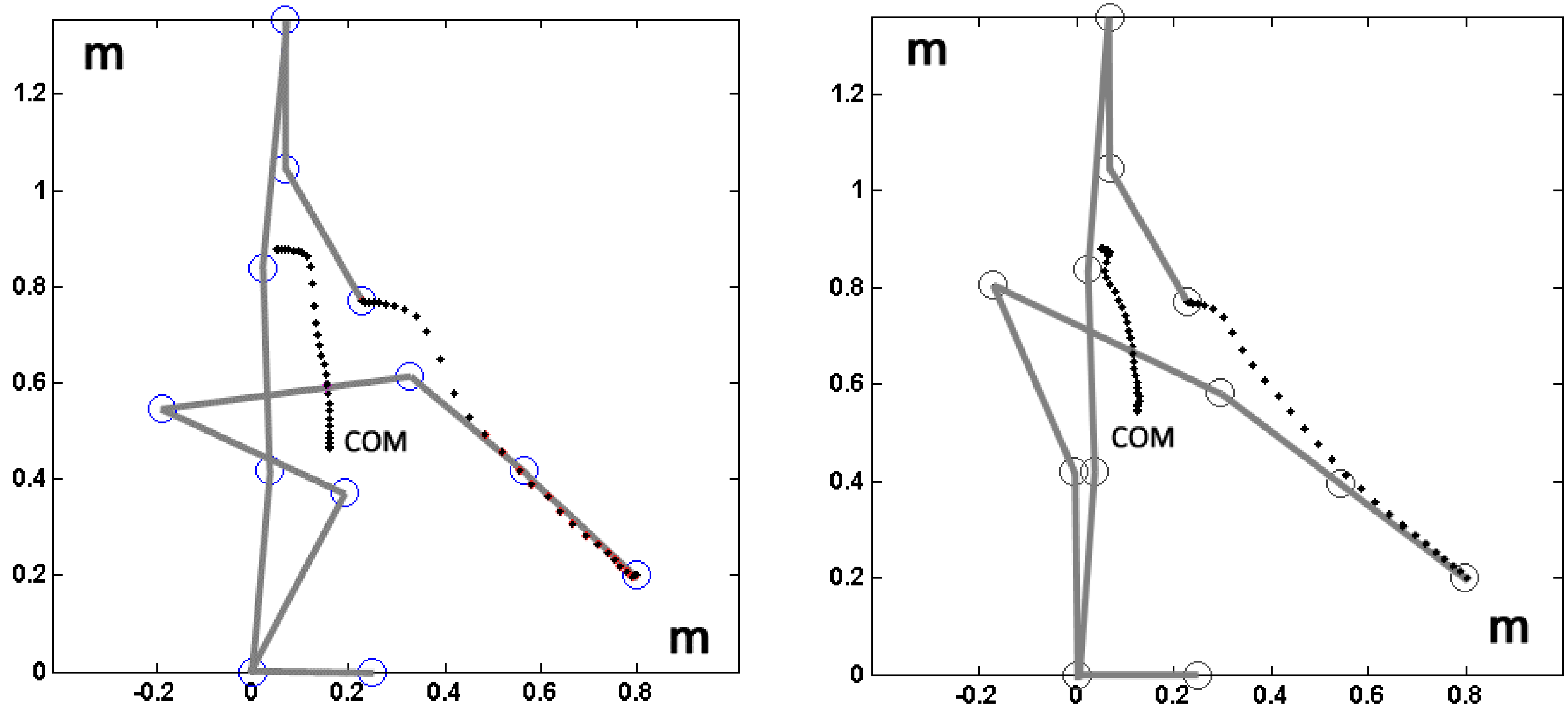
Morasso P, Casadio M, Mohan V, Zenzeri J (2009) A neural mechanism of synergy formation for Whole Body Reaching. Biol Cybern, *accepted for publ*

The COM moves forward together with the finger and the corresponding speed profiles are synchronized

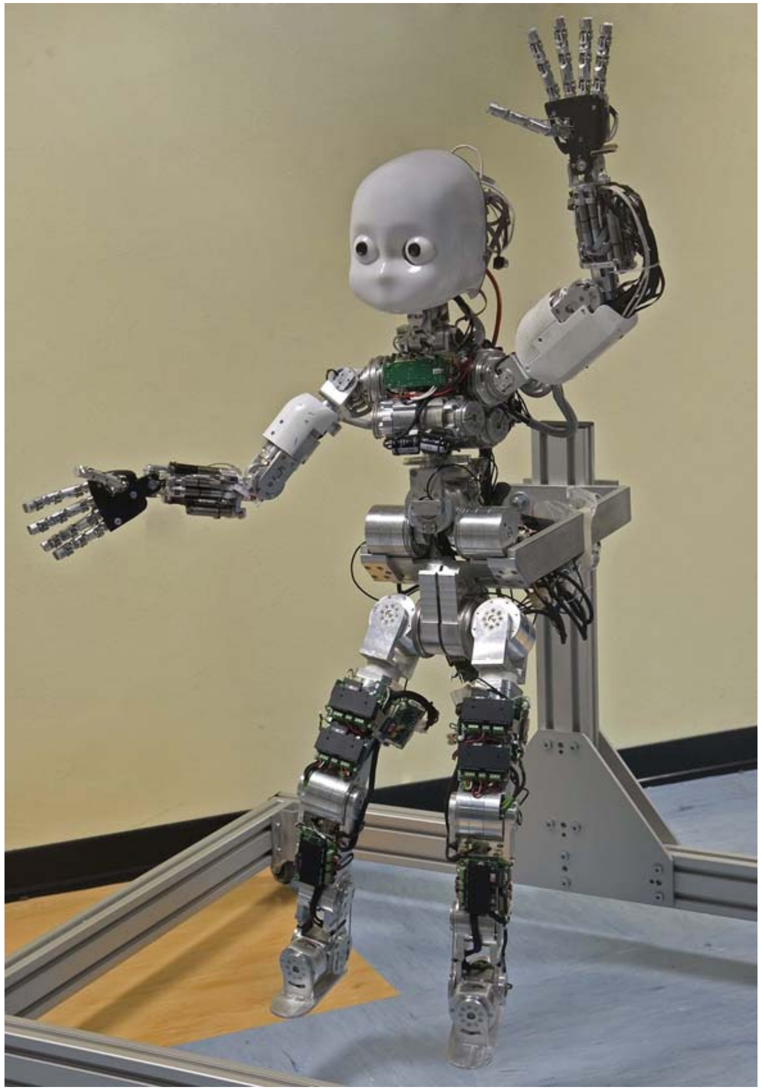


Pozzo T, Stapley PJ, Papaxanthis C (2002) Coordination between equilibrium and hand trajectories during whole body pointing movements. Exp Brain Res, 144:343–350.

The Admittance matrix ***A*** selects the virtual motion in the null space of the kinematic transformation or UCM (uncontrolled manifold)

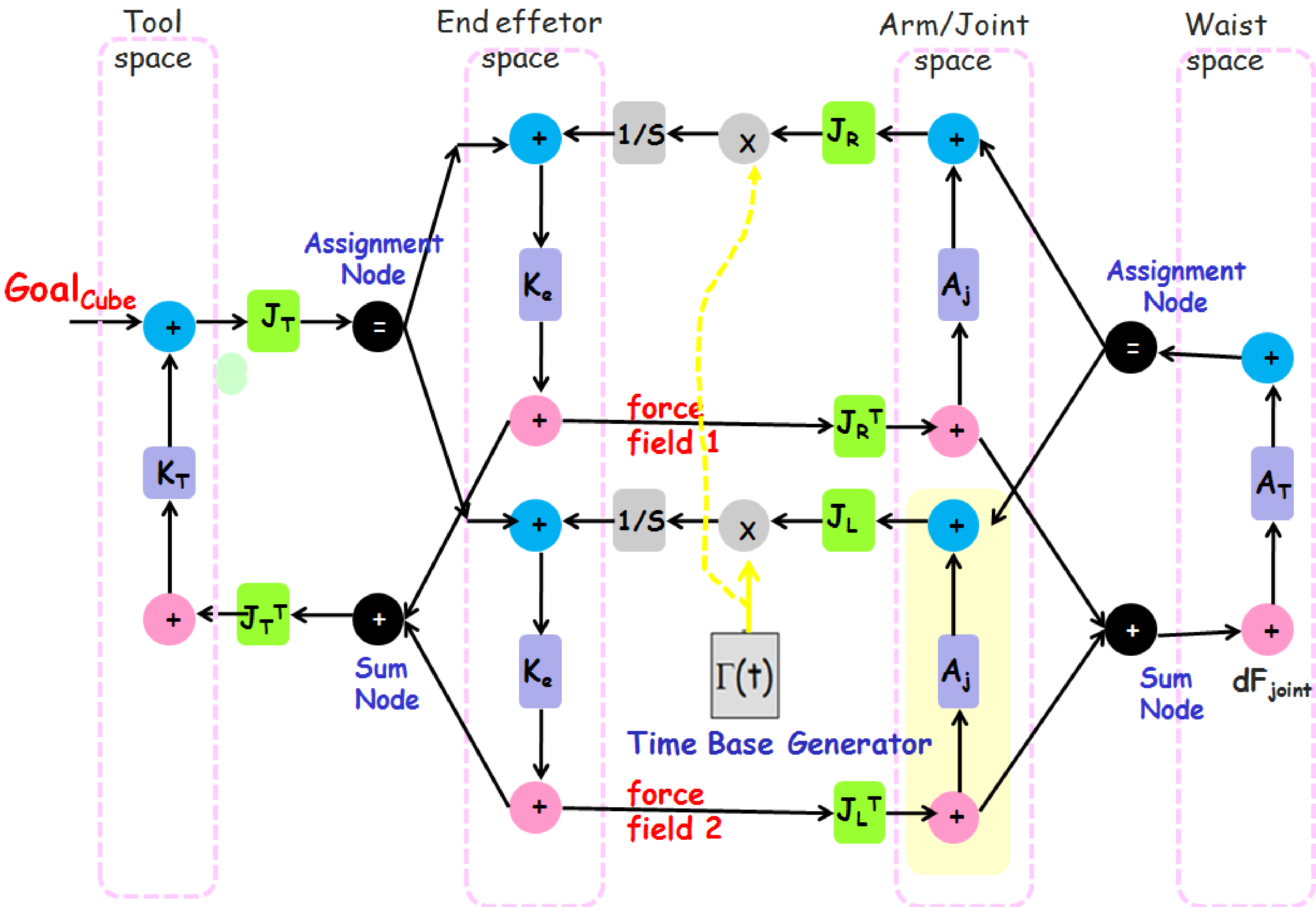


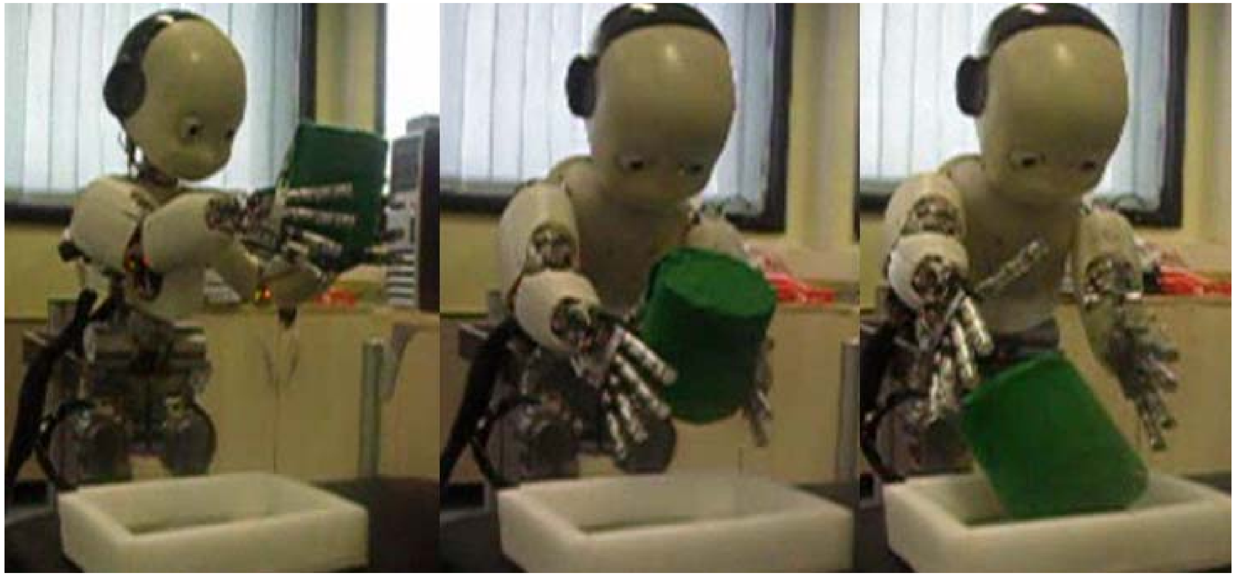
Synergy formation in a Humanoid Robot

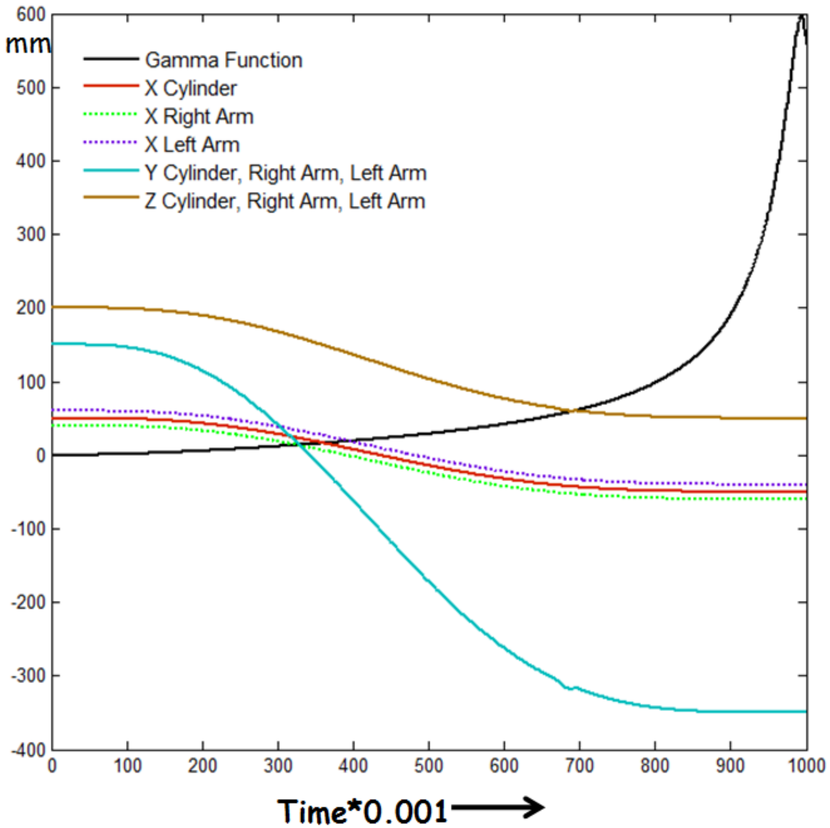


53 dof's iCub

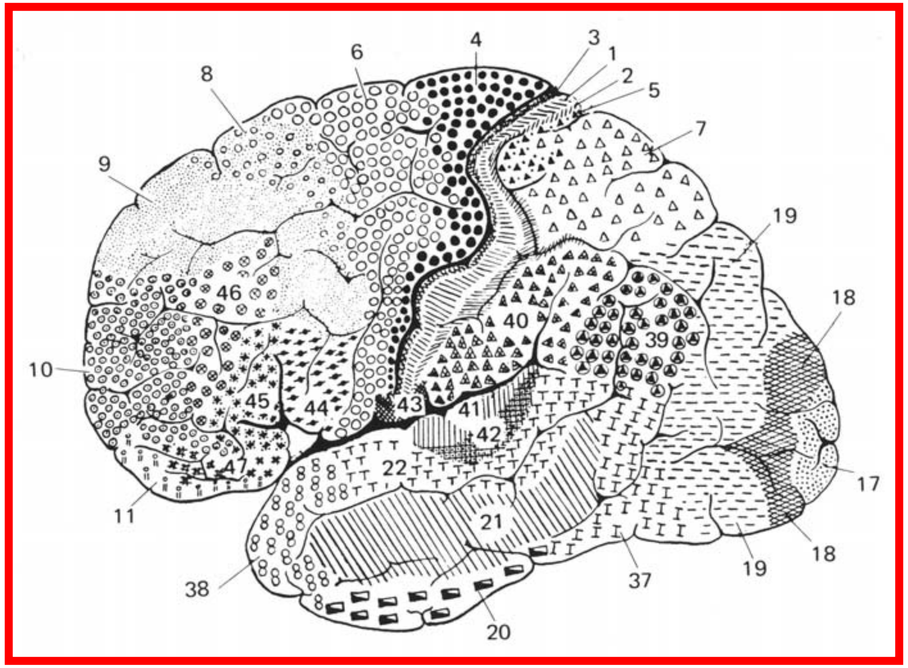
Application of PMP to iCub for bimanual coordination






600
mm
500
400
300
200
100
0
-100
-200
-300
-400
Gamma Function
X Cylinder
X Right Arm
X Left Arm
Y Cylinder, Right Arm, Left Arm
Z Cylinder, Right Arm, Left Arm
0
100
200
300
400
500
600
700
800
900
1000
Time*0.001

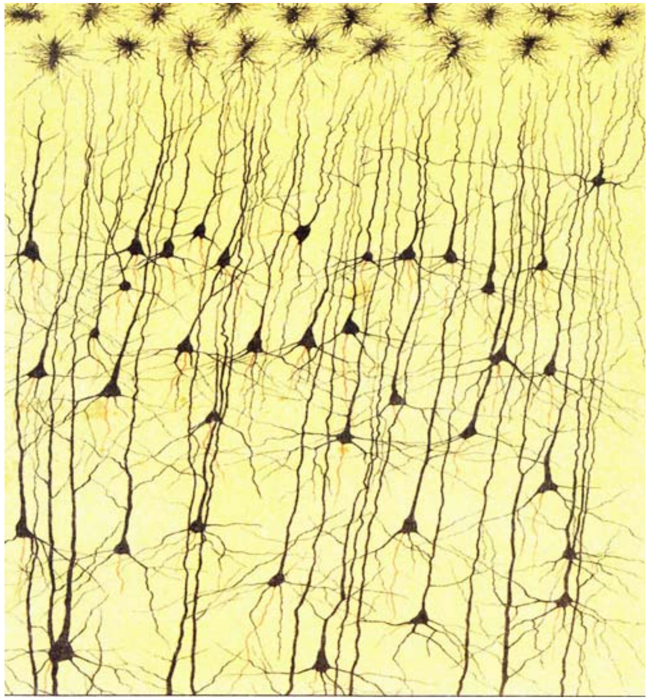
Cortical maps as topology-representing neural networks applied to motor control

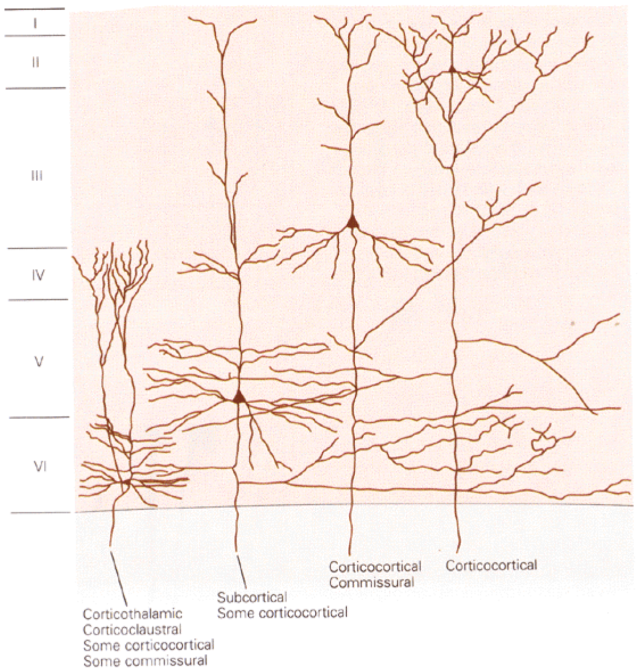


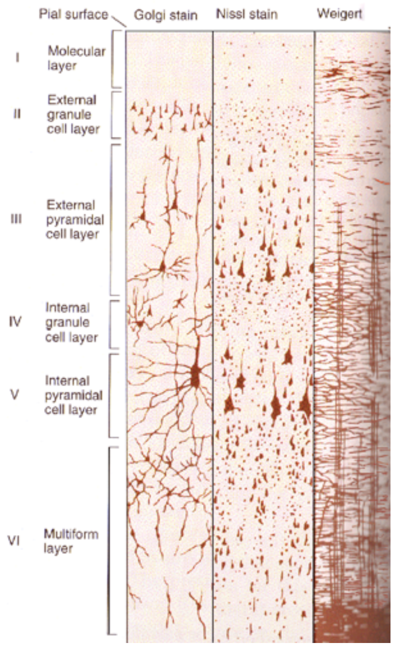
A uniform cytoarchitecture
(horizontal: layers, vertical: columns)

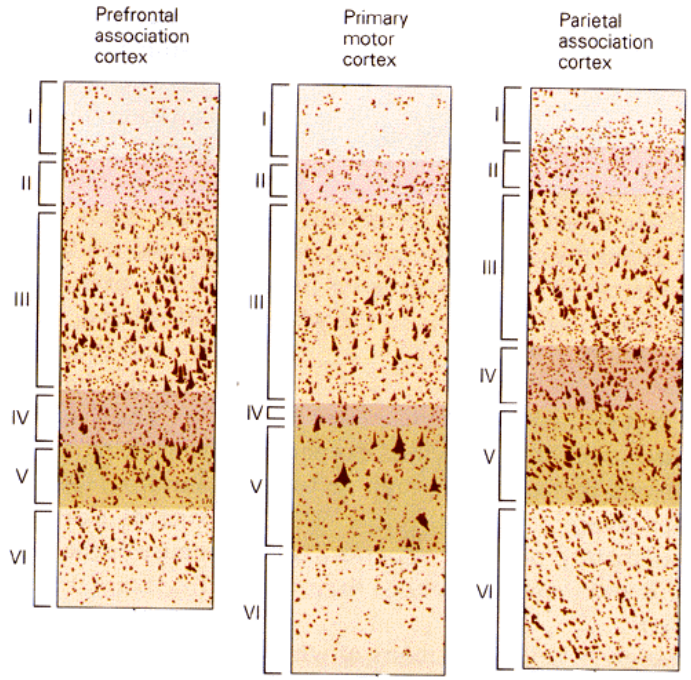
A uniform *pattern of lateral connectivity*, dominated by excitatory *reverberant connections* global dynamics of cortical areas or field-computing

Topological structure of lateral cortical connections, with inter-cluster distances typical of different maps.

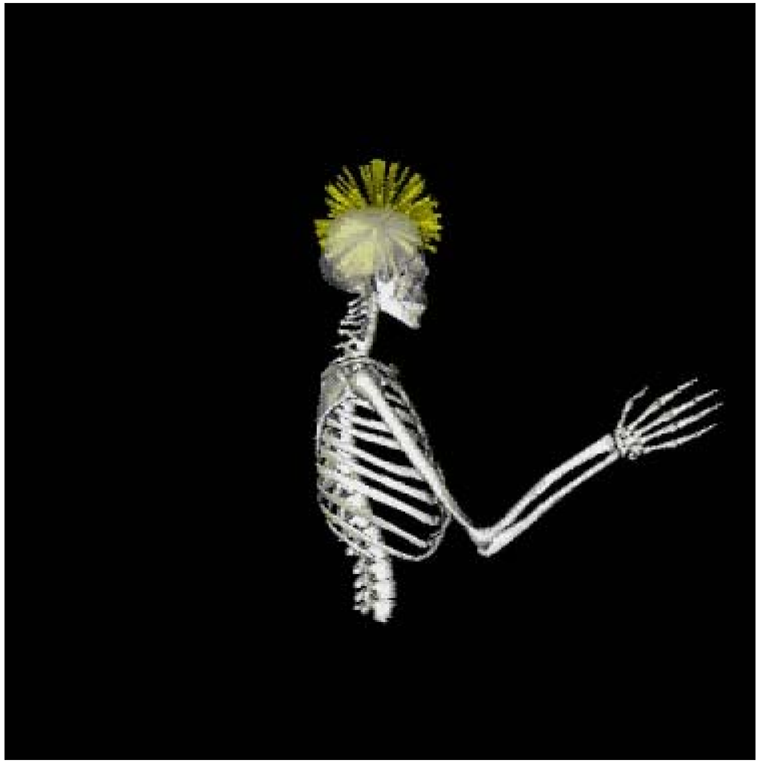








Cortical maps – *population code dynamics*



Georgopoulos, A., Schwartz, A., Kettner, R. (1986). Neuronal population coding of a movement direction. *Science*, 233, 1416-1419

Mappe auto-organizzanti

- Dinamica competitiva
- Apprendimento Hebbiano non supervisionato
- Reticolo di connessioni laterali

Famiglia di modelli riconducibile ad un paradigma comune, ispirato alle ***mappe corticali biologiche***:

- SOM (self-organizing map): T. Kohonen
- NG (neural gas): T. Martinetz
- Soft competitive learning
- ….

Il termine mappa si riferisce al fatto che i vettori contenenti i pesi di connessione dei neuroni sono visti come punti su una ipersuperficie (manifold) ospitata in un “feature space”.

Il termine auto-organizzante implica che la “mappatura” o “tassellazione” viene trovata in modo non-supervisonato (Hebbian learning), mediante una “dinamica competitiva”.

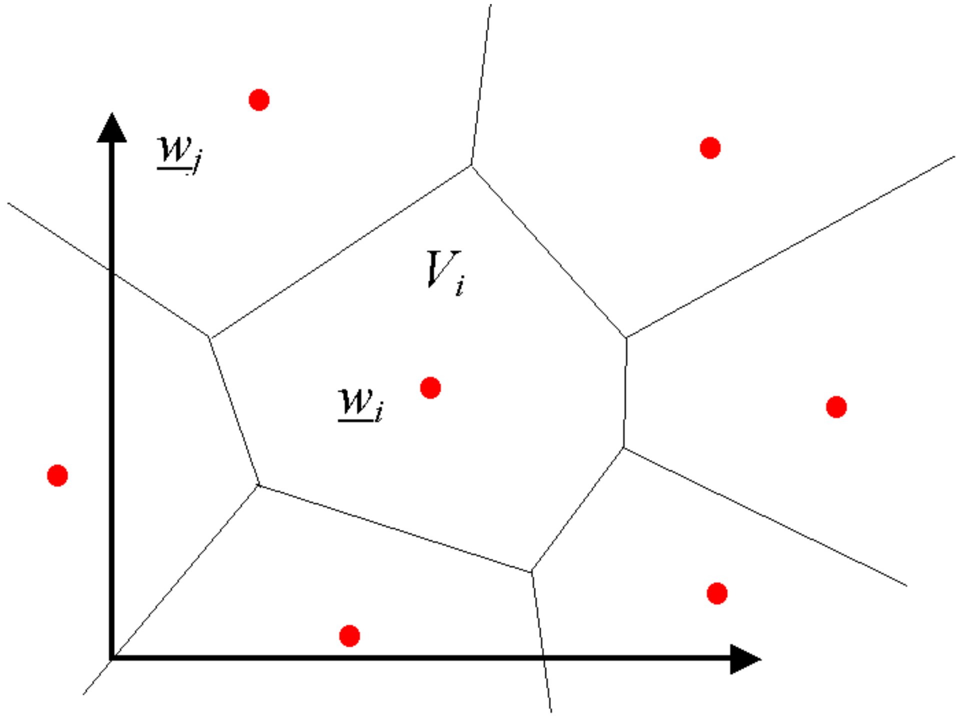
Un modello auto-organizzante può essere visto come un *GRAFO* definito da *nodi* (i neuroni) e *connessioni laterali*.

Ogni nodo è caratterizzato, in ogni istante, da: un vettore prototipo (centro del *Campo Recettivo*) + un *Livello di attività* (che dipende dallo stimolo di ingresso e dal campo recettivo)

Un codebook C induce in M una *TASSELLAZIONE DI VORONOI* :

ad ogni prototipo $\underline{w}_i$ corrisponde un volume V_i che è un elemento della tassellazione del manifold M

$$\boldsymbol{M = \sum_i V_i}$$



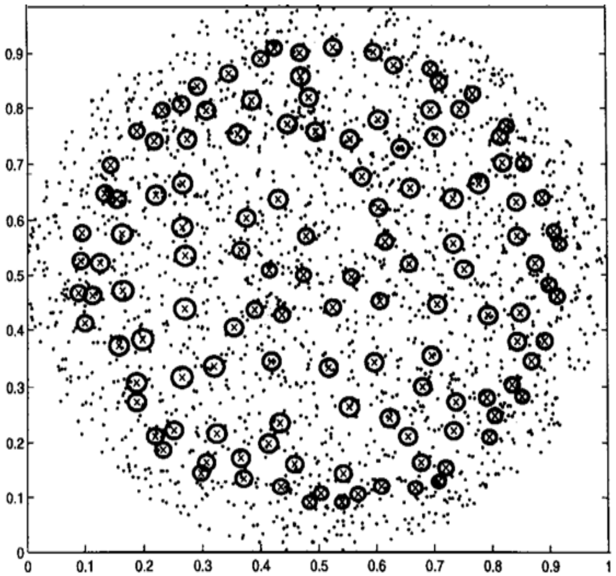
La minimizzazione di Q è un problema complesso perché Q dipende da C in modo altamente non-lineare

In una buona tassellazione ogni elemento V_i contiene lo stesso numero di campioni di $D \Rightarrow$ L'entropia del "codice ottimo" è massima.

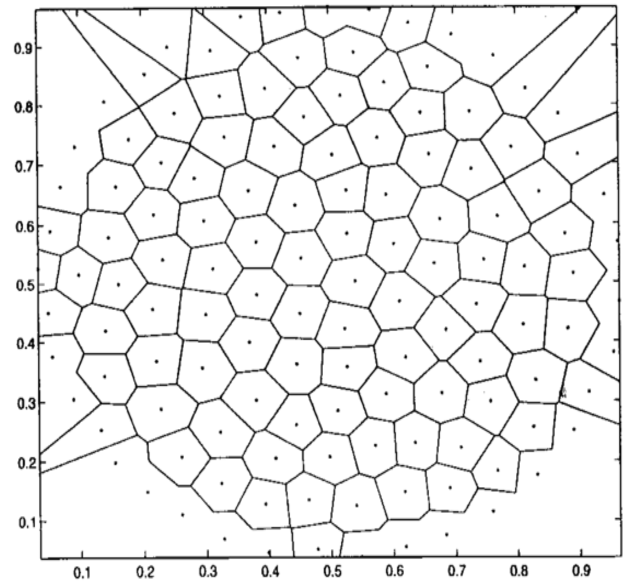
QUANTIZZAZIONE VETTORIALE

tassellazione di *VORONOI* e triangolazione *di DELAUNAY*

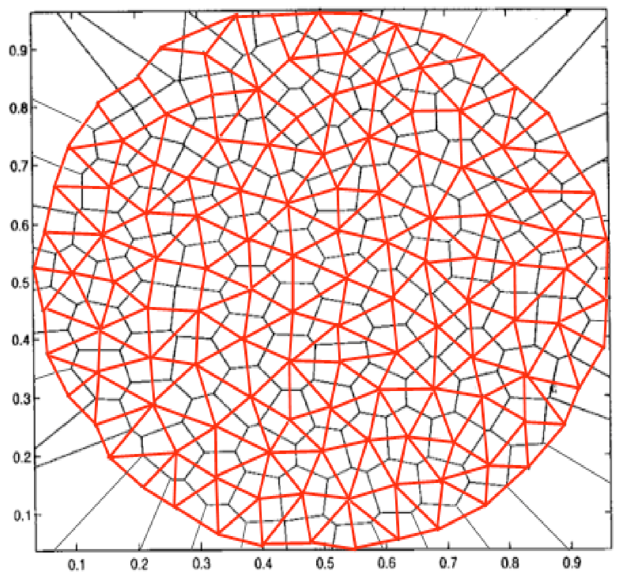
Dati e prototipi



Tassellazione di Voronoi



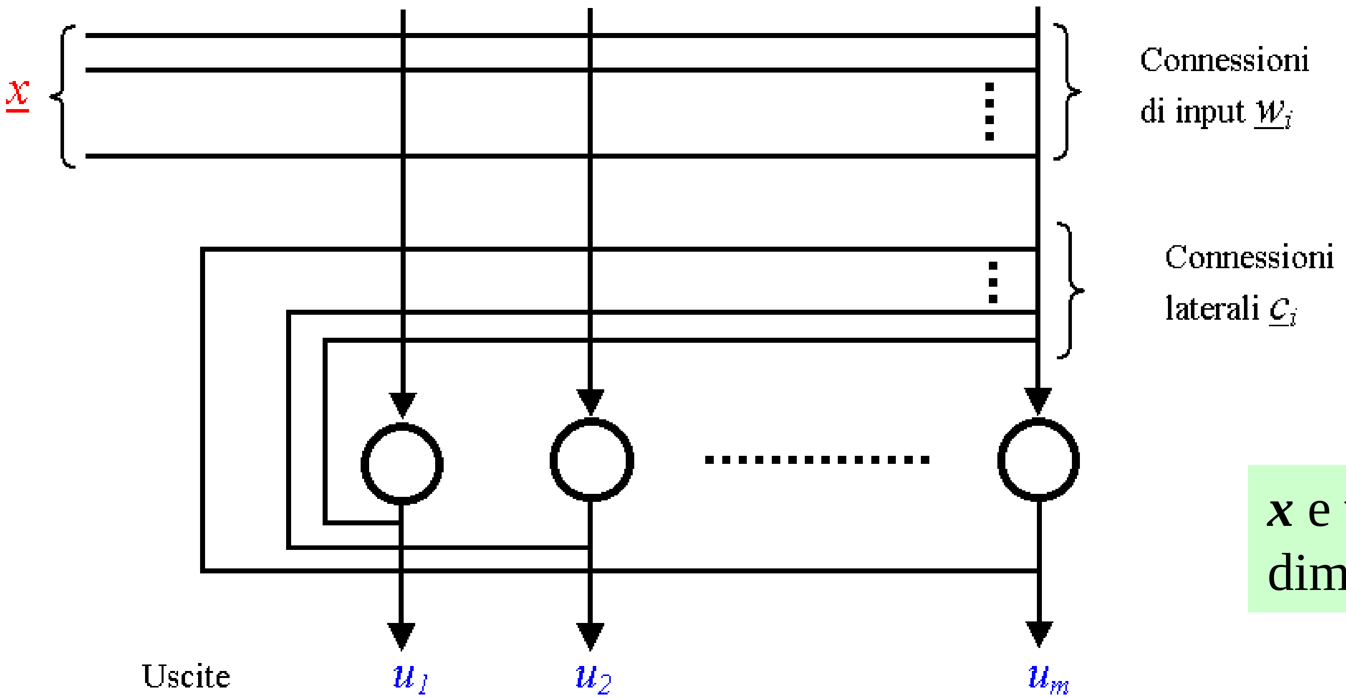
Tringolazione di Delaunay



Triangolazione di Delaunay: grafo o reticolo in cui ogni prototipo è “collegato” con i prototipi le cui regioni di Voronoi hanno almeno una faccia in comune con la regione di Voronoi del prototipo dato

La dinamica competitiva può essere implementata:

- Winner-take-all (*inibizione ricorrente:* ogni neurone ha una connessione inibitoria con tutti gli altri tranne se stesso)
- Winner-take-most (*connessione ricorrente a Mexican-hat*: eccitazione dei vicini più prossimi ed inibizione di vicini più lontani)



Centri dei campi recettivi

x e $\boldsymbol{w_i}$ hanno la stessa dimensionalità

MAPPE AUTO-ORGANIZZANTI – famiglie di modelli (elenco non esaustivo)

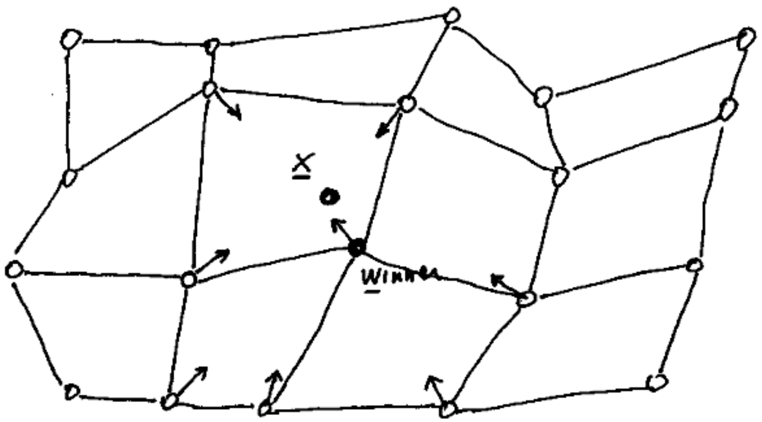
Modello	Addestramento delle connessioni laterali	Addestramento dei centri dei campi recettivi	Addestramento della dimensione dei campi recettivi	Figura di merito (da minimizzare durante l'apprendimento)
SOM	No (le connesioni sono fisse)	Hebbiano	No	Non ben definita
NG	Si (algoritmo euristico di accensione e decadimento)	Hebbiano (con guadagno che dipende dalla vicinanza topologica)	No	Non ben definita
TRN	Si (come sopra, con un ulteriore funzione di trimming)	Hebbiano (idem come sopra)	No	Non ben definita
Soft-Competitive learning	Non ci sono	Hebbiano (con guadagno che dipende da un ordinamento nello spazio degli stimoli)	No	Cross-entropy
EM (Expectation maximization)	Non ci sono	E-step + M-step	Si	Log-likelihood

Nelle SOM la topologia delle connessioni laterali è prefissata (1-D, 2-D …)

L'apprendimento è di tipo Hebbiano:

$$\Delta \underline{w}_i = \eta(\underline{x} - \underline{w}_i)u_i(\underline{x})$$

dove η è il "guadagno" e la formula implica che i centri dei campi recettivi si "muovono" in direzione dello stimolo corrente $\underline{x}$ proporzionalmente al livello di attivazione u_i :



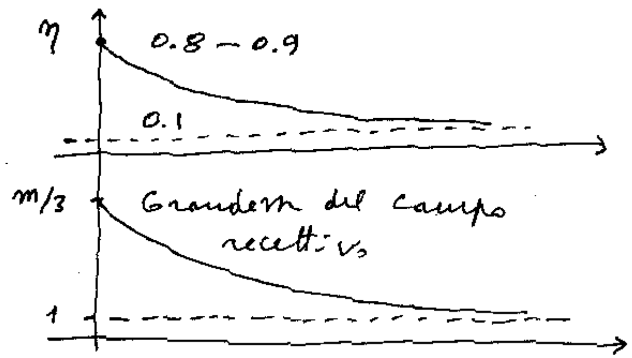
Mappe di Kohonen: diversi tipi di connessioni laterali

Fig. 5.14. Distribution of weight vectors, rectangular array

Fig. 5.15. Distribution of weight vectors, linear array

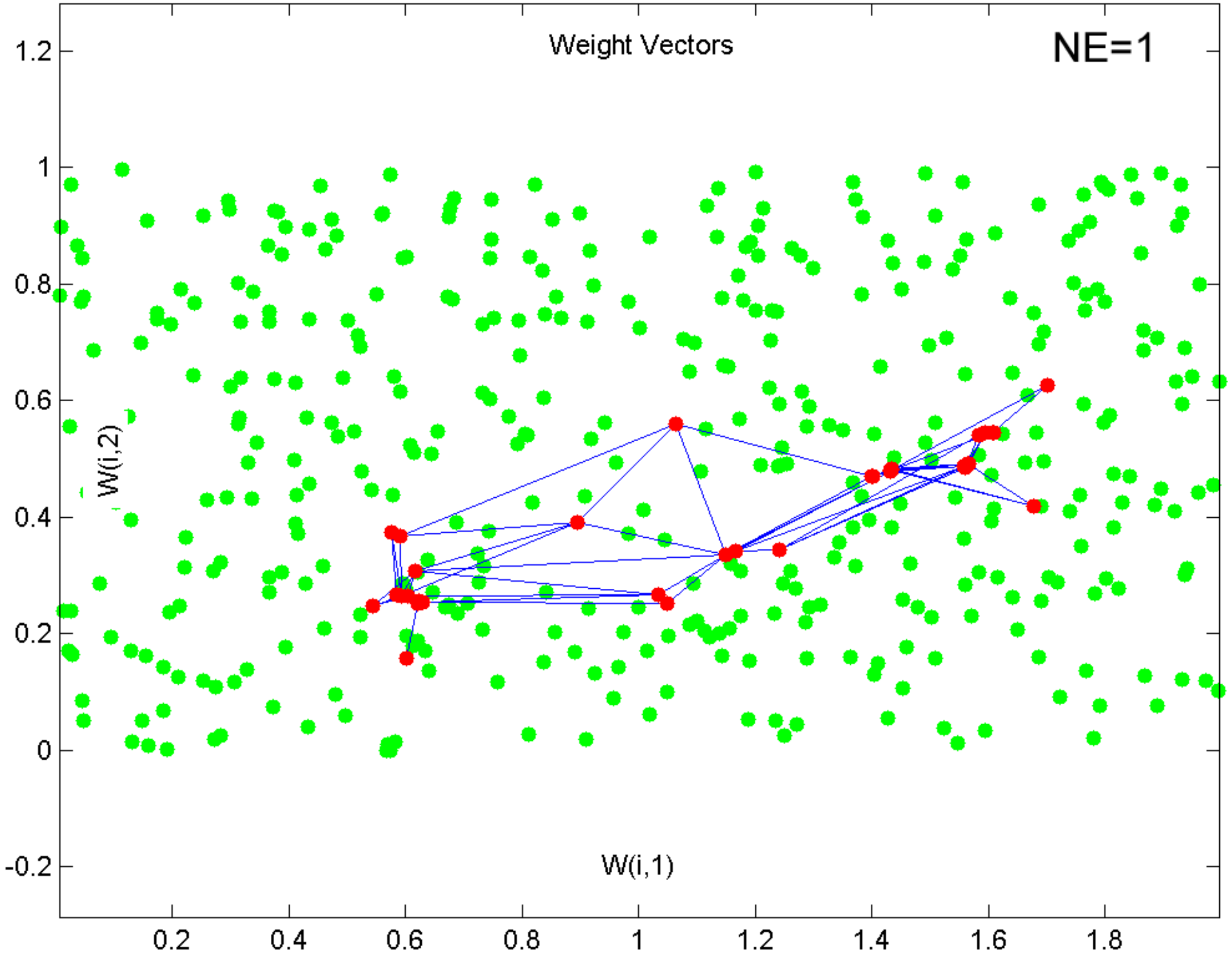
Fig. 5.13. Distribution of weight vectors, hexagonal array

Empiricamente si è verificato che per una buona riuscita del training sia η sia la dimensione del campo recettivo devono essere modulate in discesa, costituendo una specie di simulated annealing:

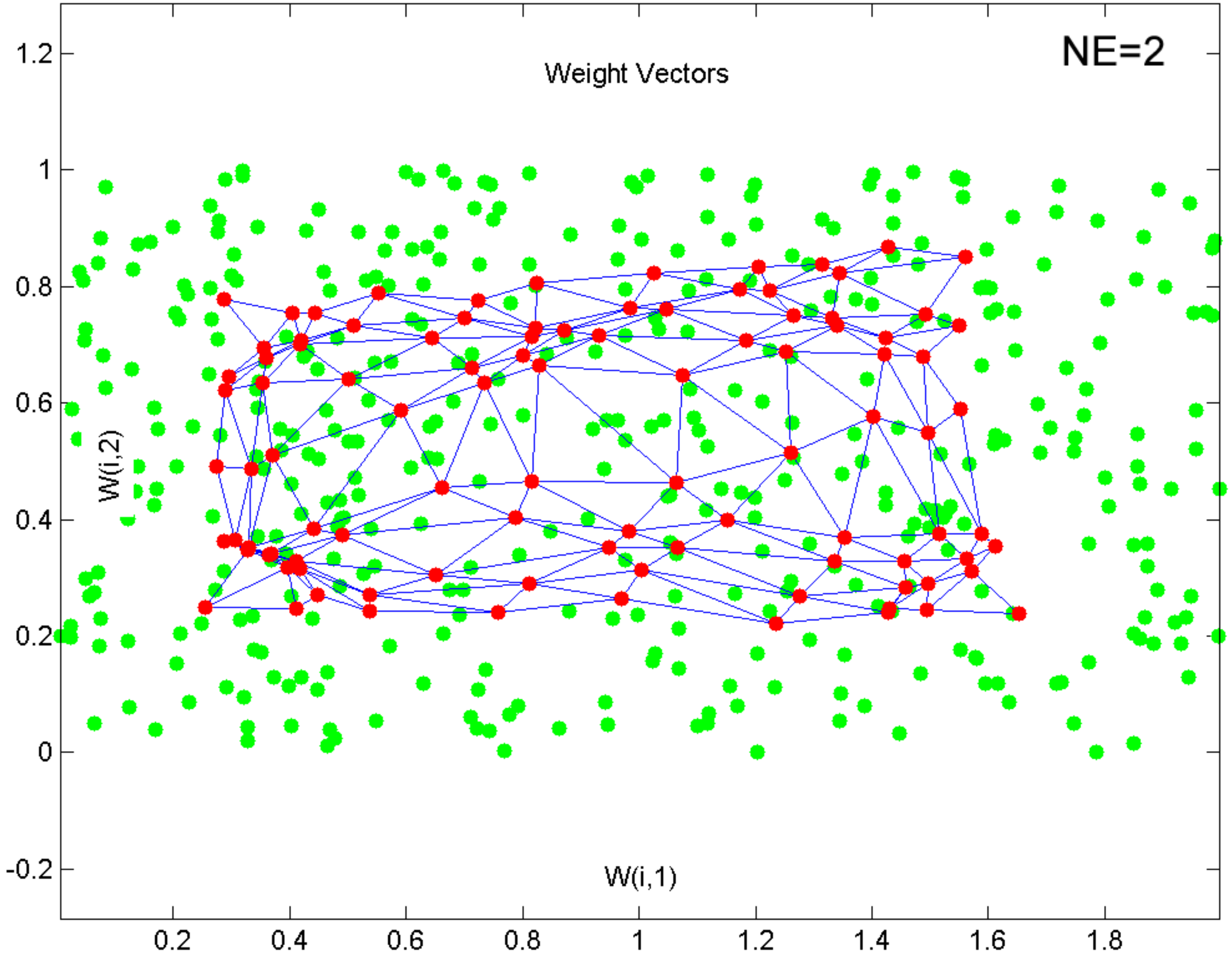


NE: number of epochs

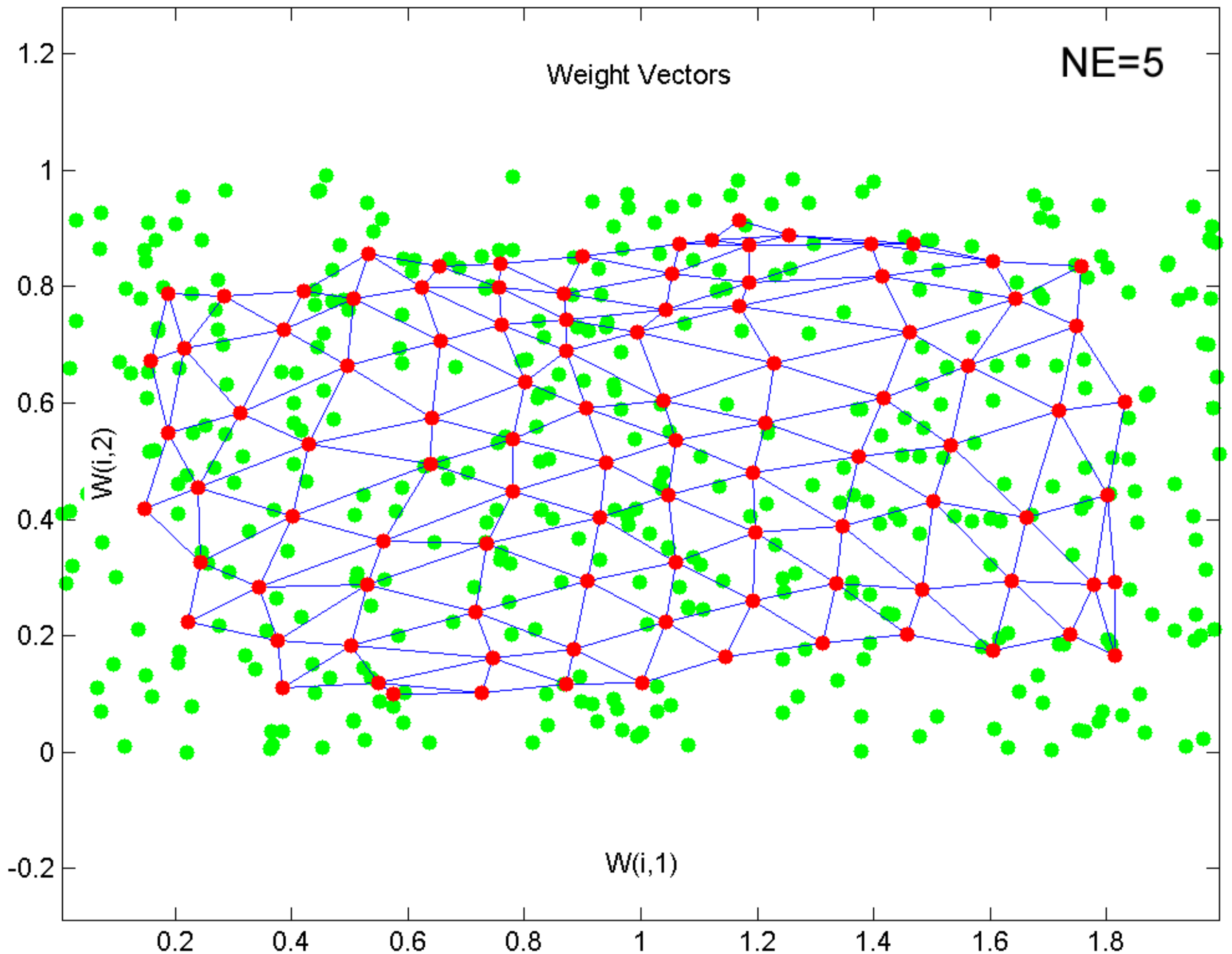
Epoch: Hebbian learning with all the elements of the training set, randomly selected



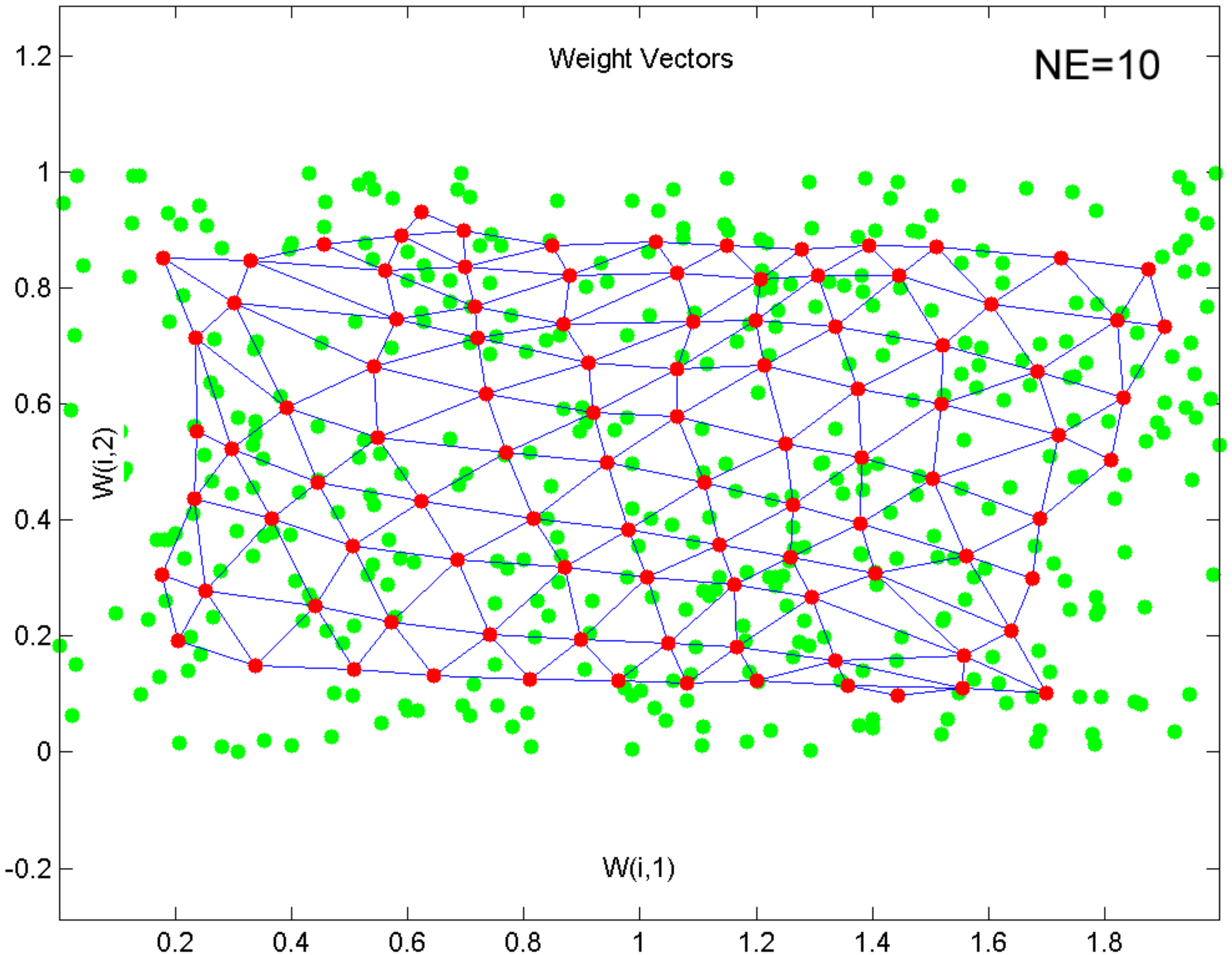
SOM model: the number of neurons is fixed a priori and the grid of lateral connection is also chose a priori (in this case hexagonal grid)

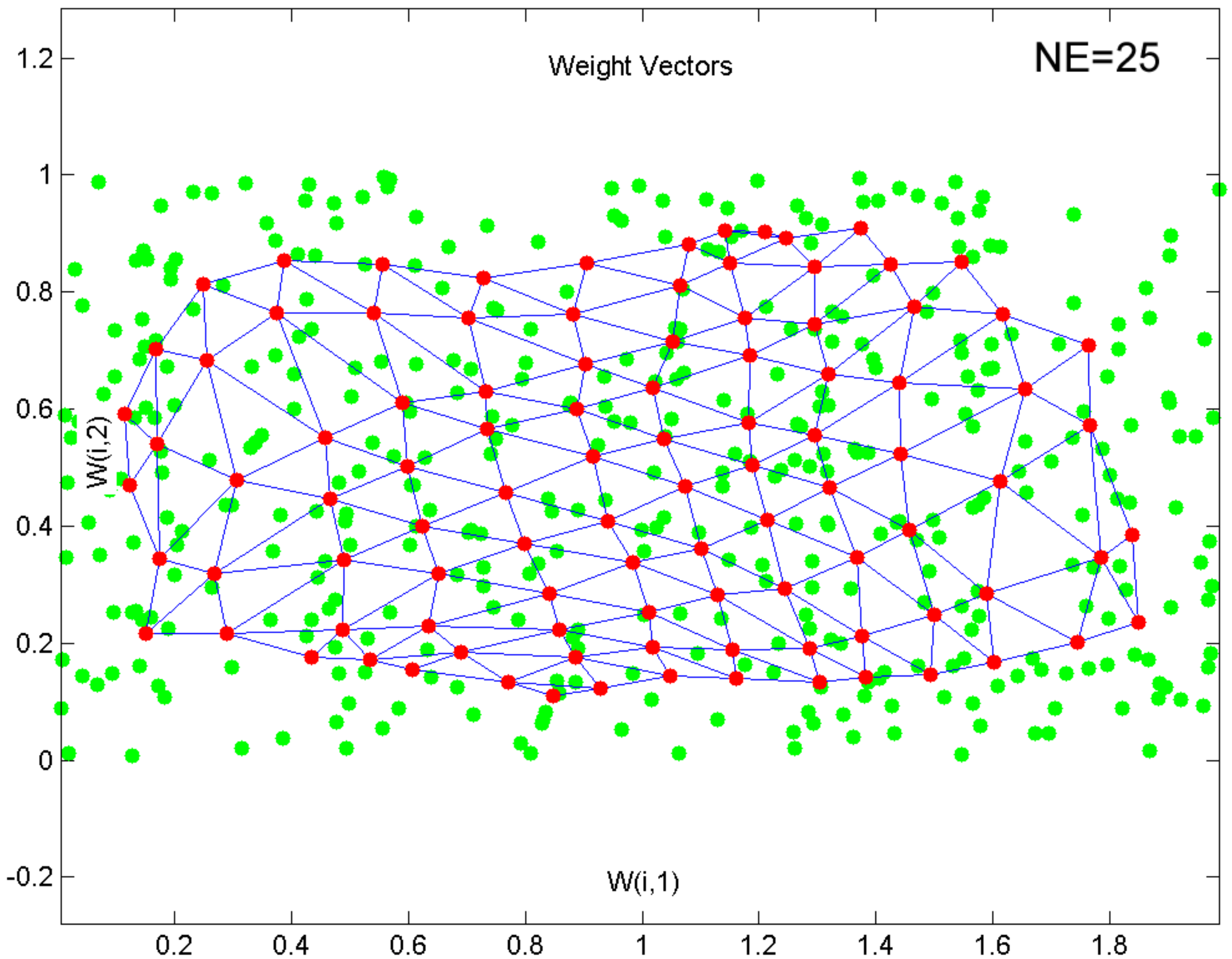


Public domain software http://www.cis.hut.fi/research

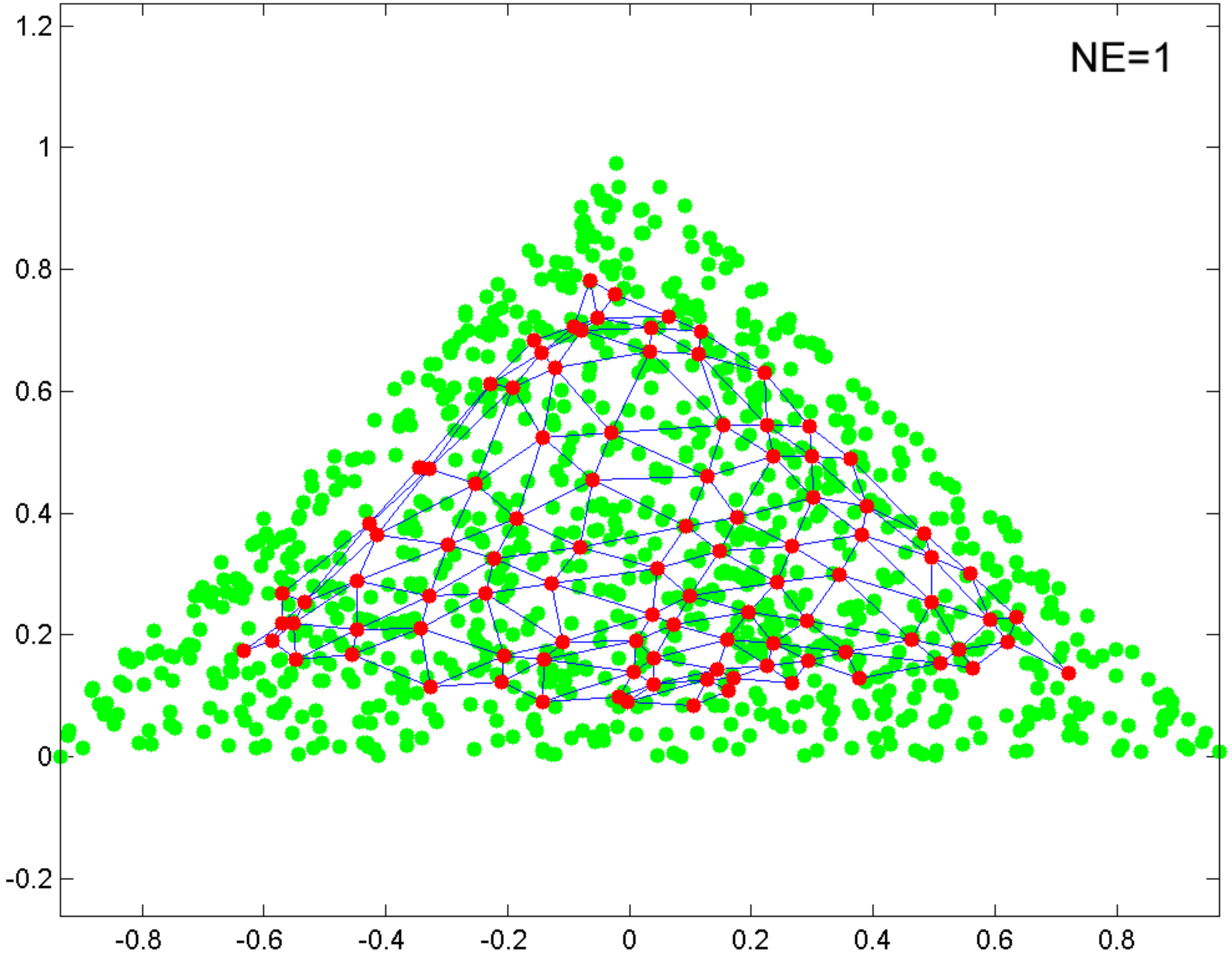


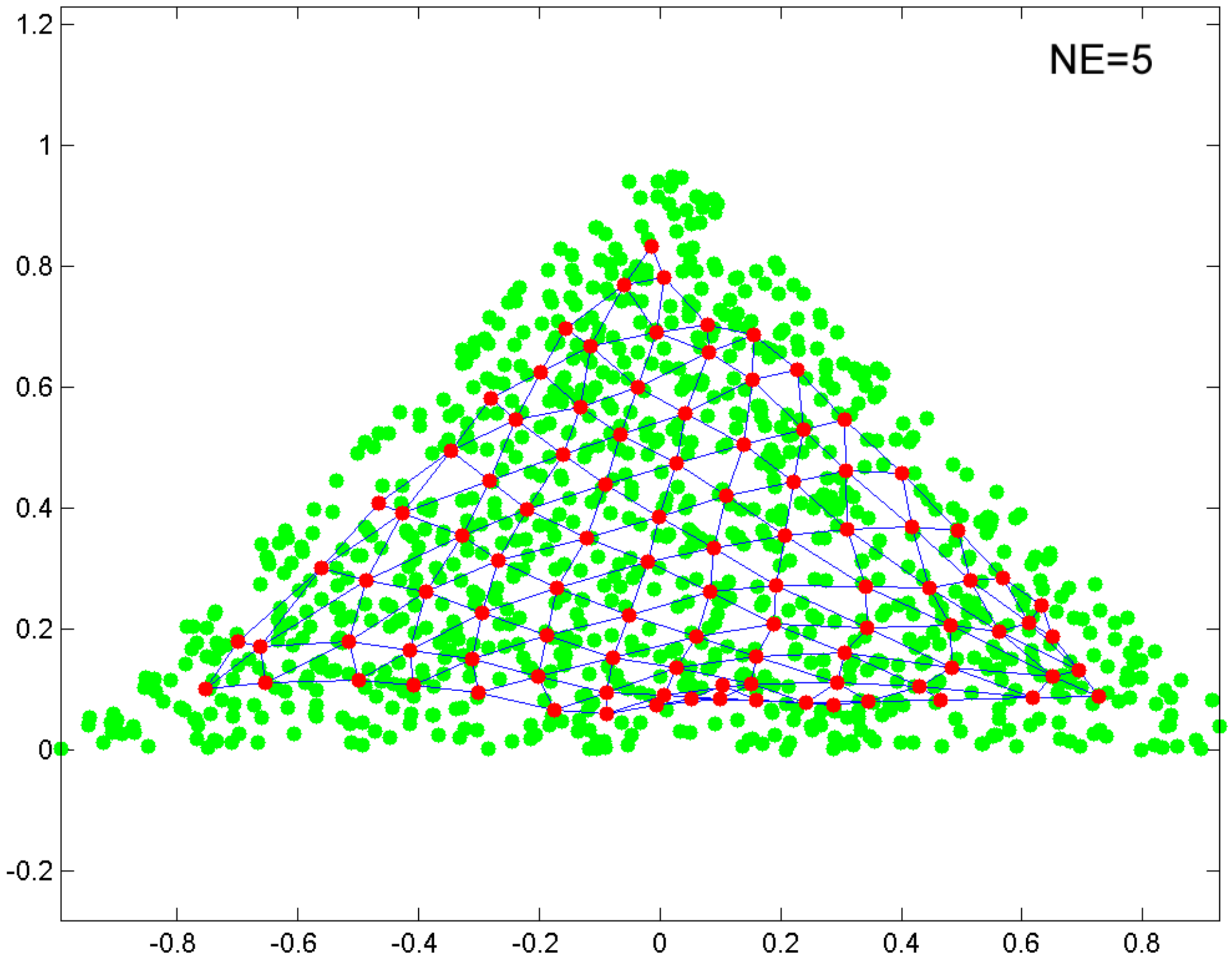
Weight Vectors
NE=5
W(i,2)
W(i,1)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

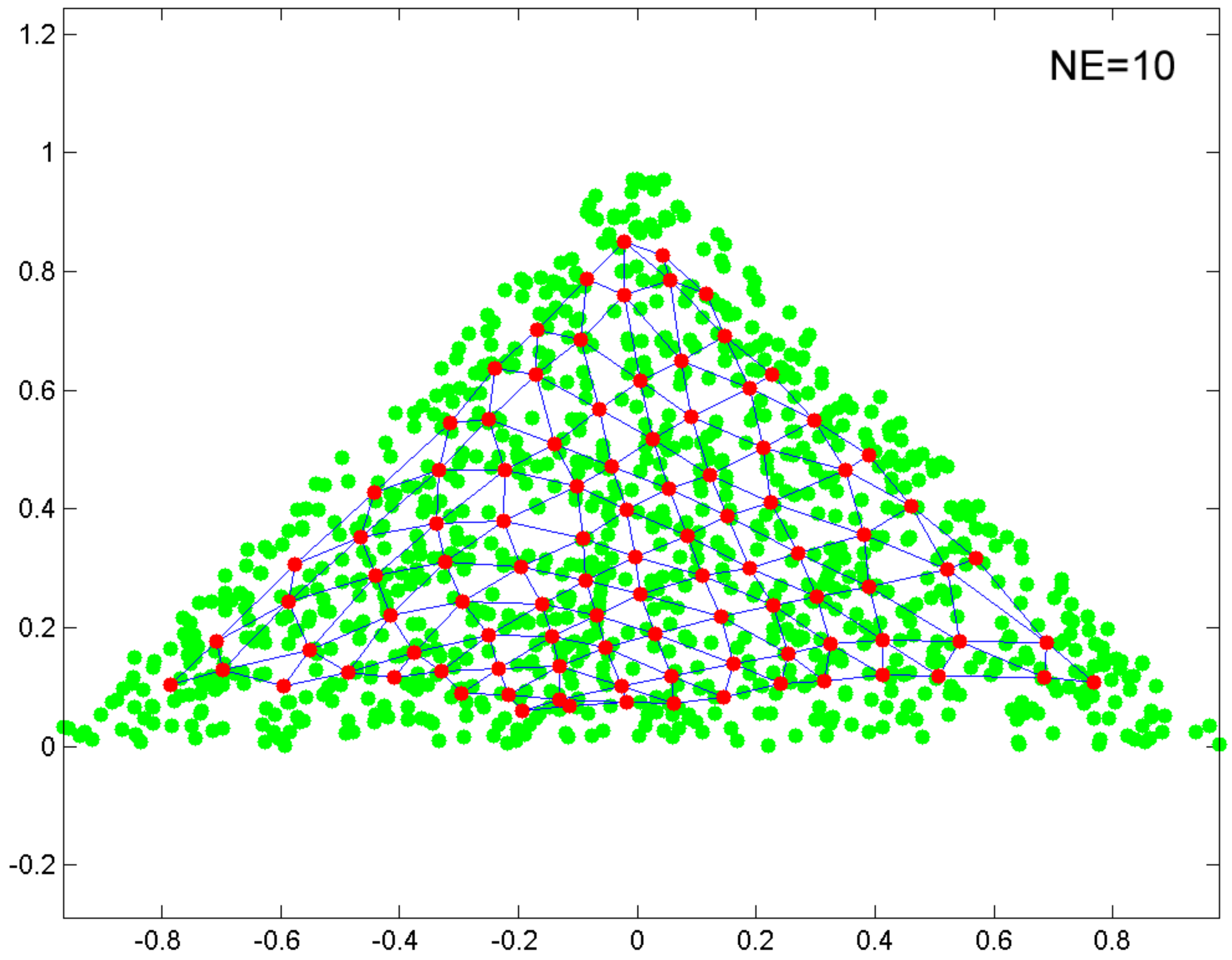

Weight Vectors
NE=10
W(i,2)
W(i,1)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8



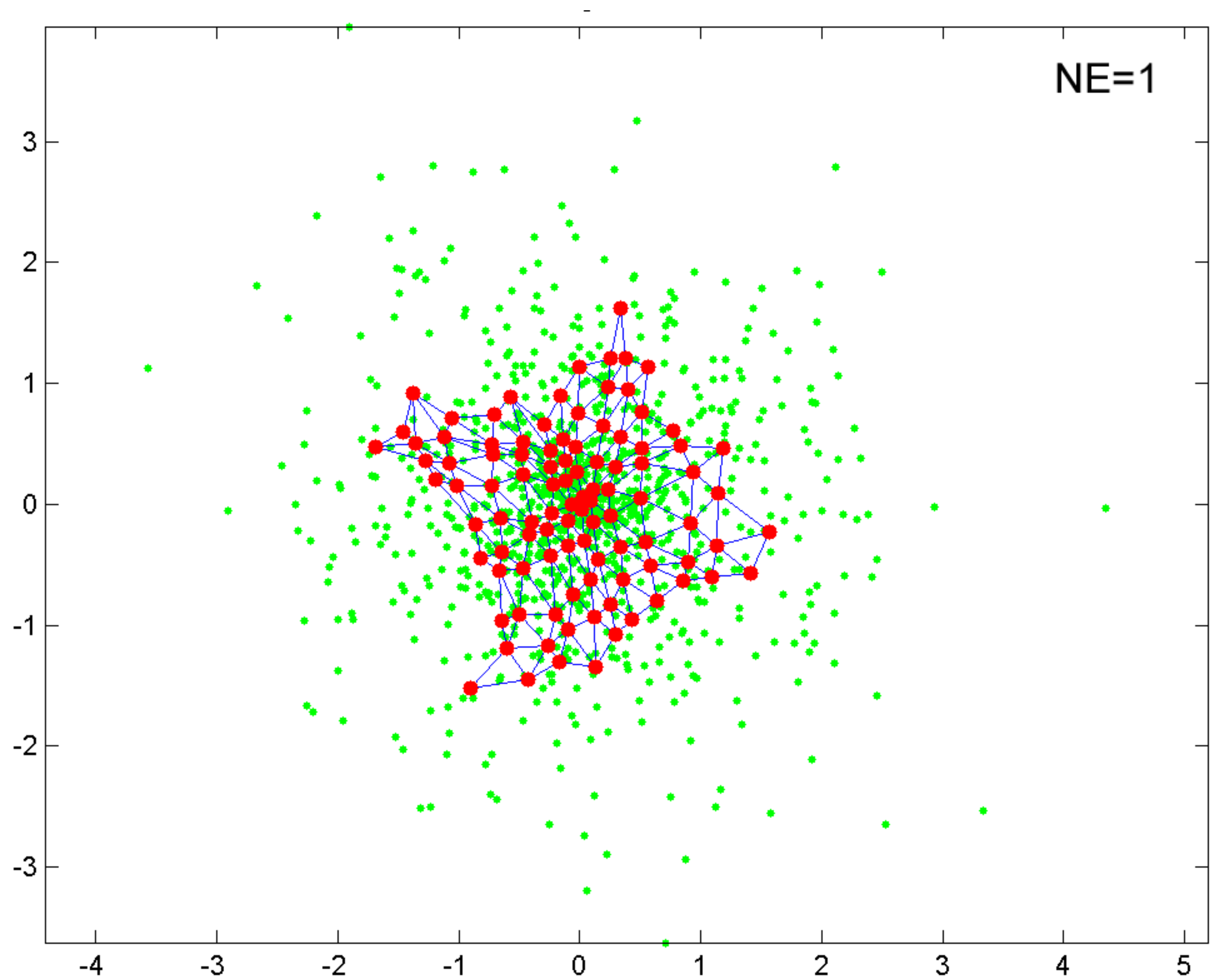
Weight Vectors
NE=25
W(i,2)
W(i,1)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

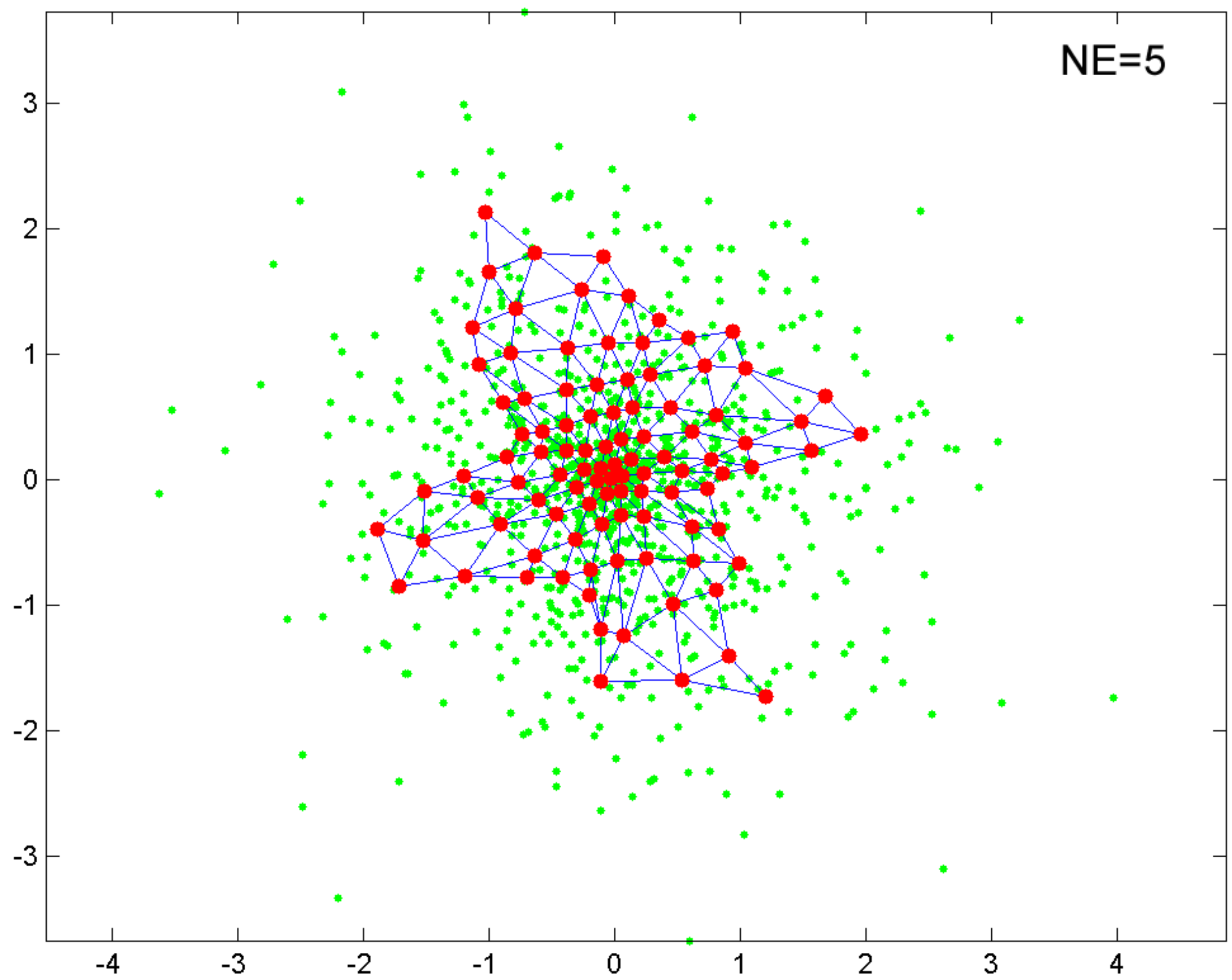

NE=1
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

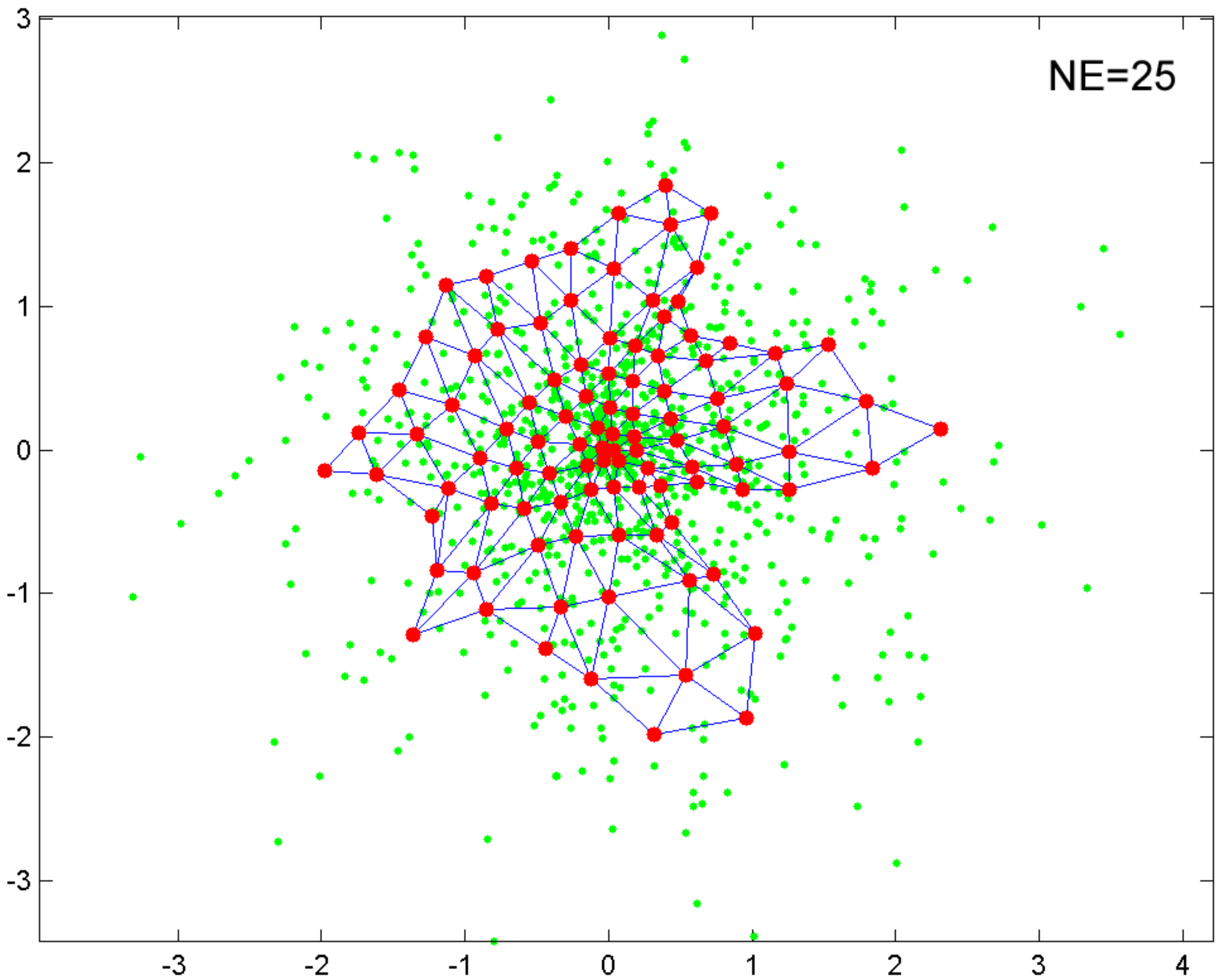

NE=5
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8



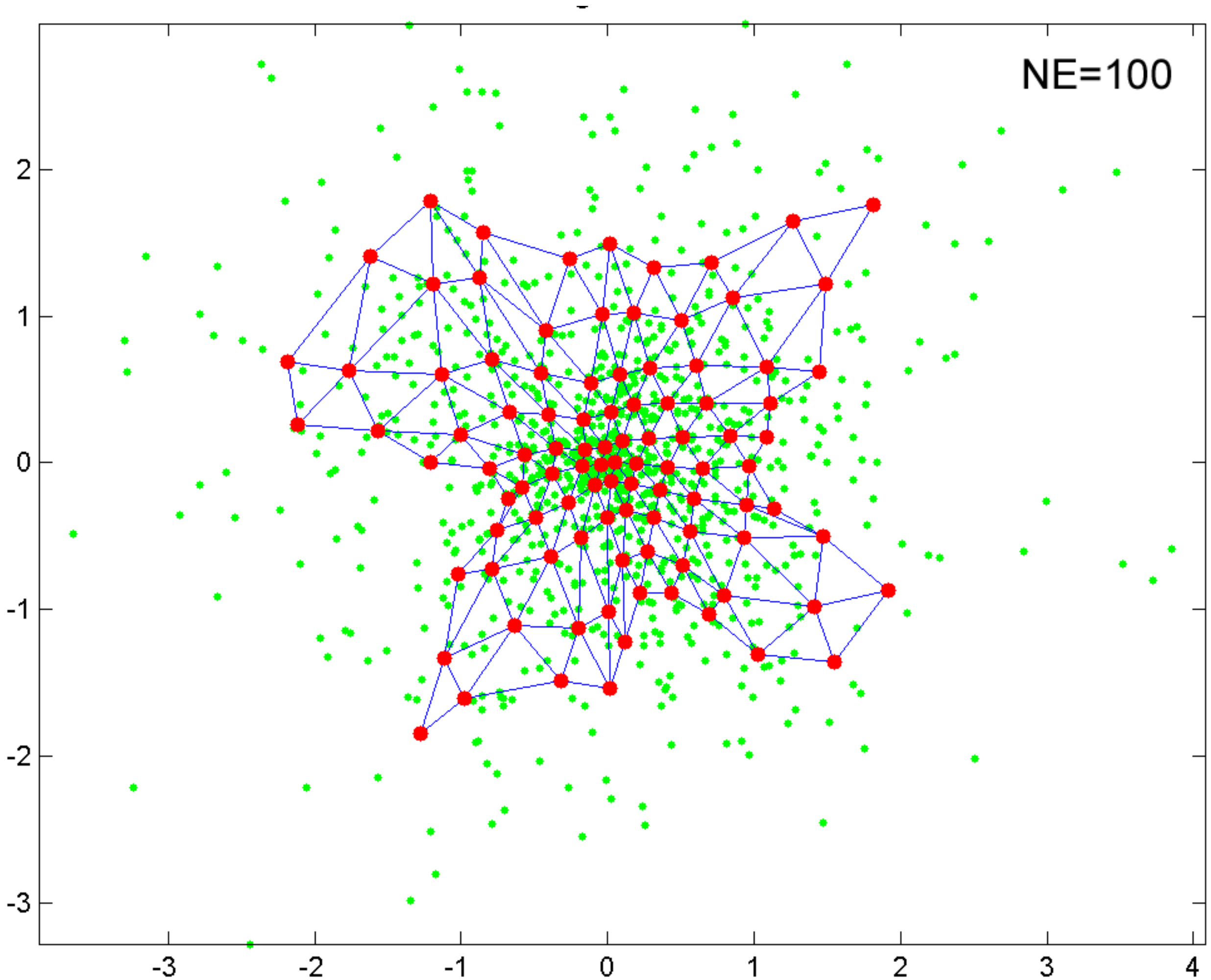
NE=10
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

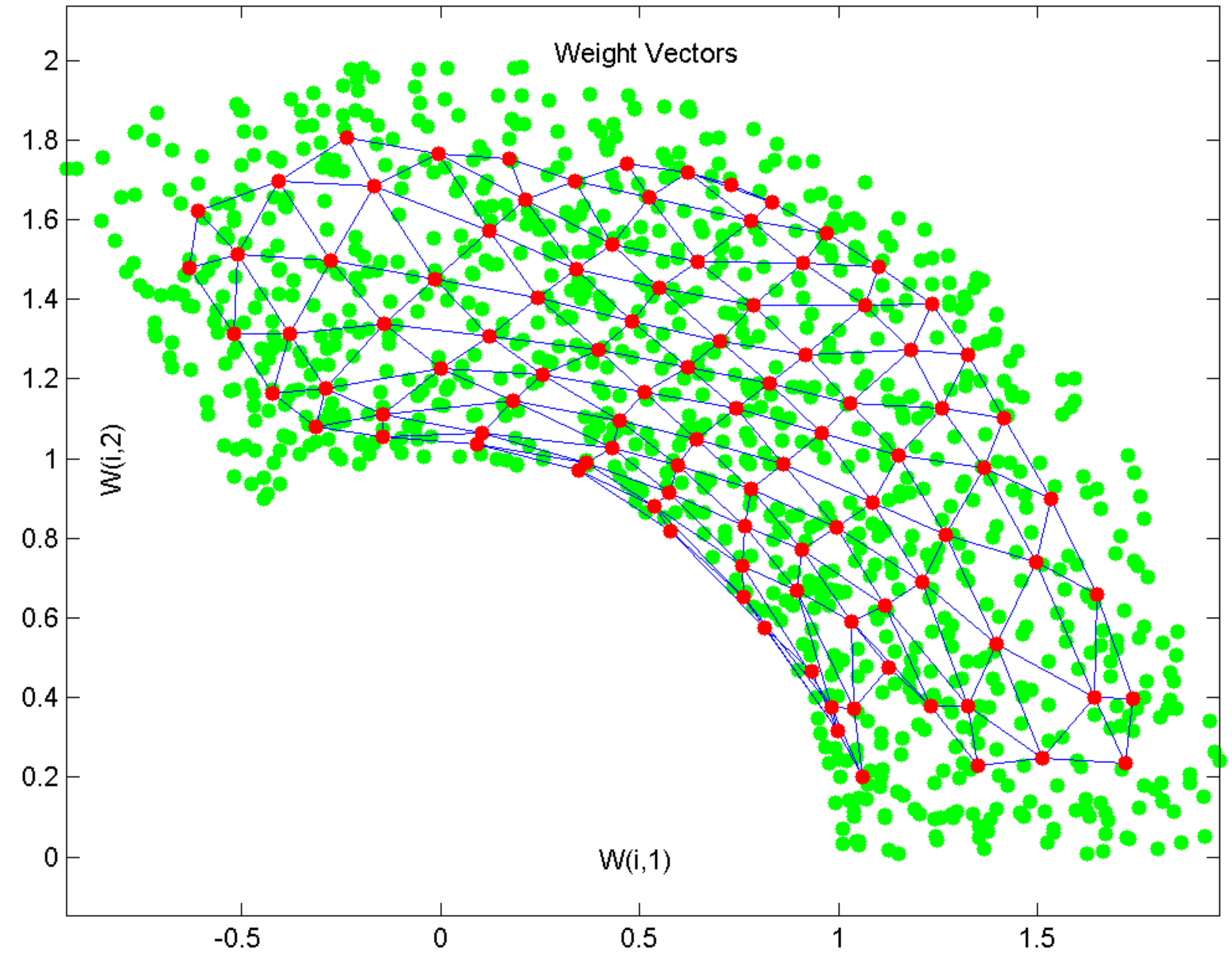

NE=1
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4
5


NE=5
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4

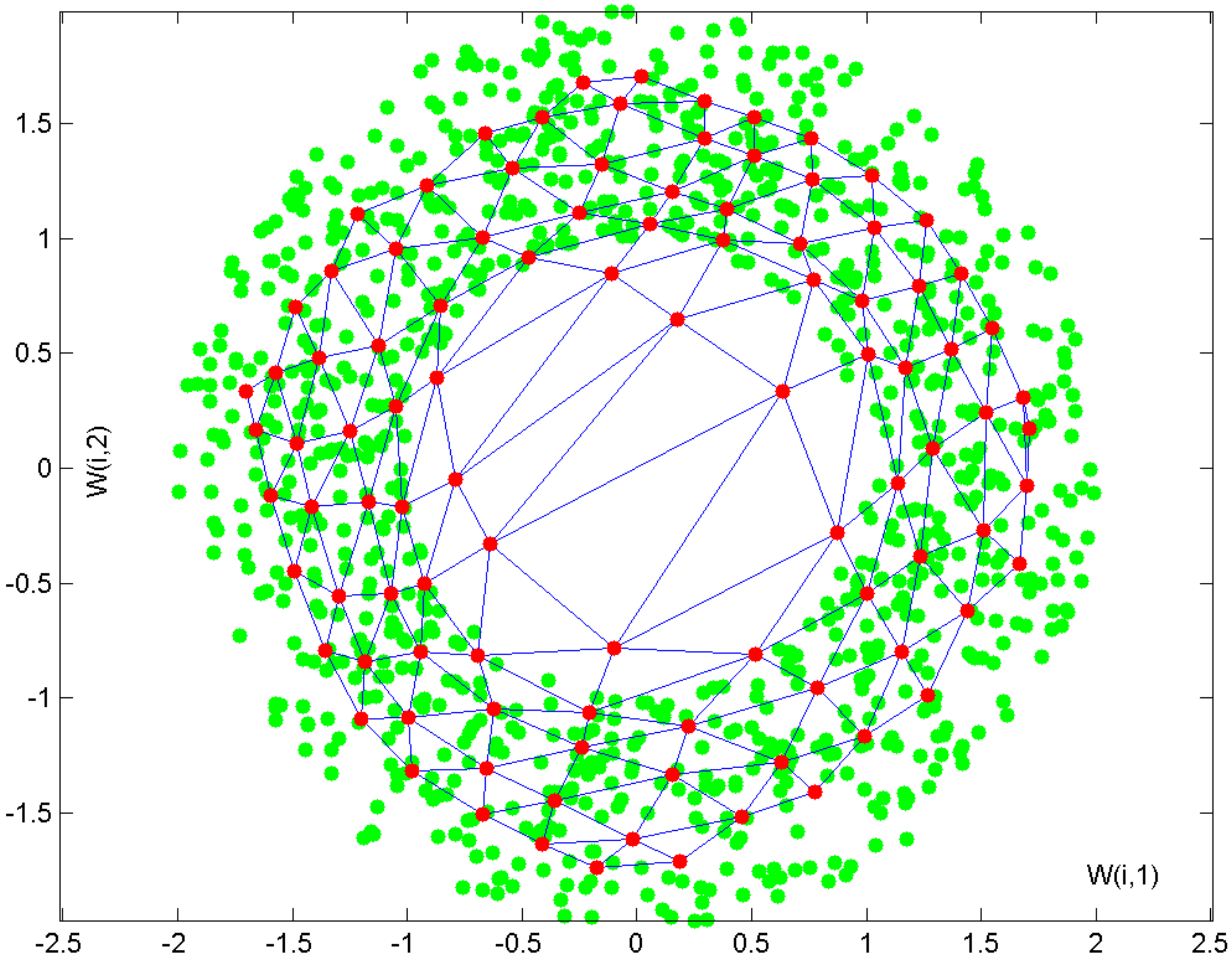


NE=25
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3
4


NE=100
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3
4

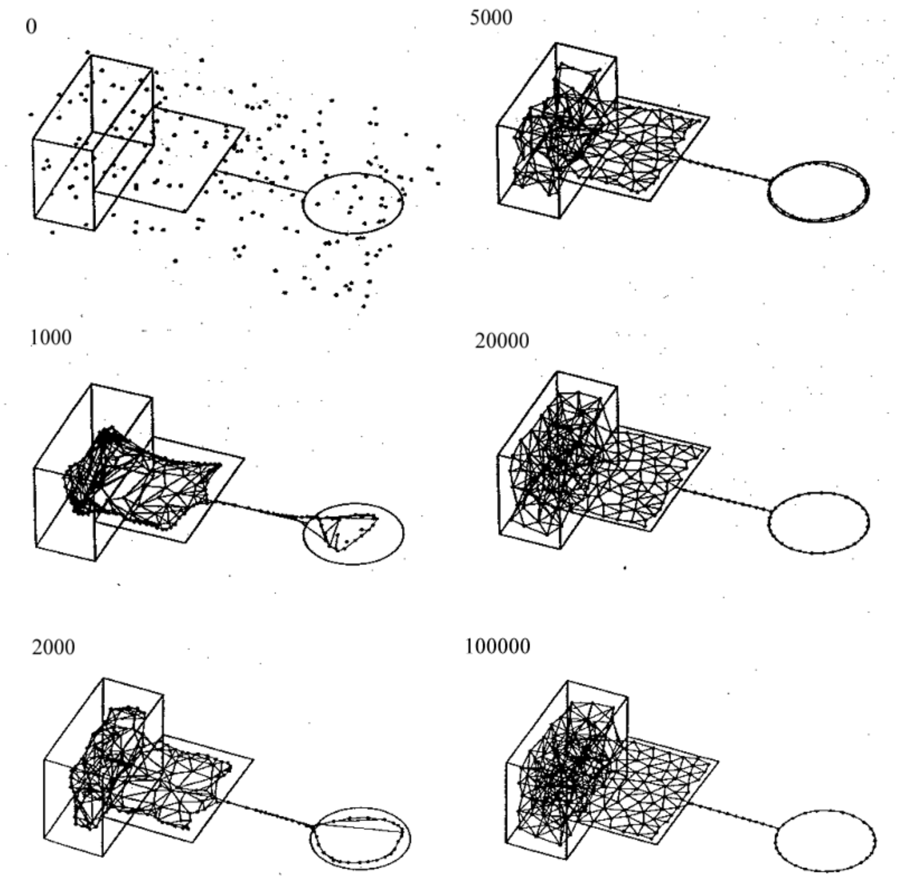

Weight Vectors
W(i,2)
W(i,1)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.5
0
0.5
1
1.5

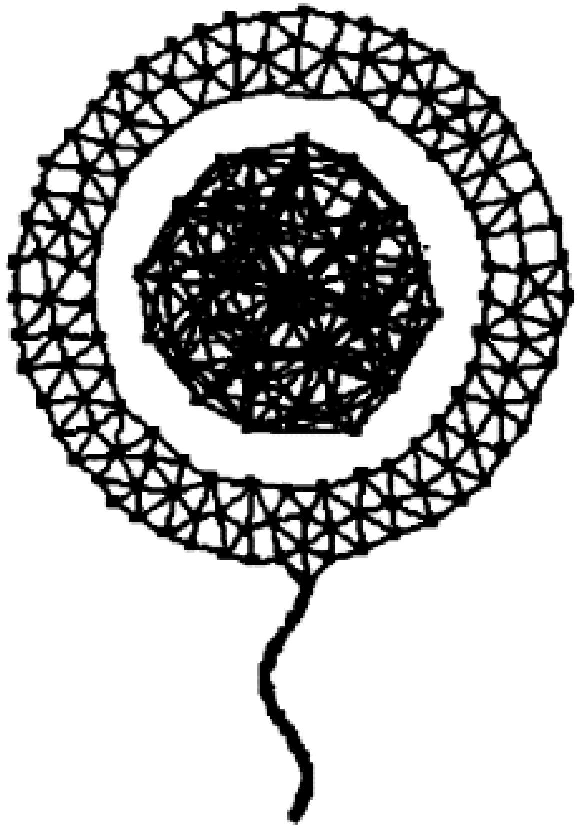
SOM models can only represent well convex manifolds.



For number non-convex manifodls it is necessary to use a competitive learning mechanism also for the lateral connections.

NG: Neural Gas (Martinez & Schulten)





Modello di mappa corticale – *population code dynamics*

Mappa: insieme di micro-colonne

V_i il livello di attività di una microcolonna

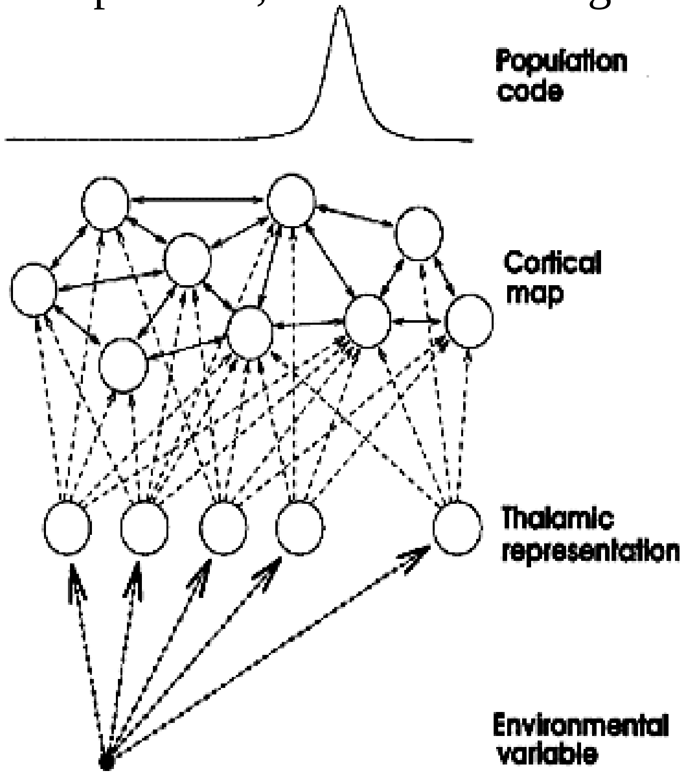
C_{ij} connessioni laterali simmetriche

W_{ik} connessioni talamo-corticali o cortico-corticali

Unsupervised, Hebbian learning

$$\frac{dV_i}{dt} = g(t)\left[-\gamma_i V_i + \Sigma_j C_{ij} \frac{V_j}{\Sigma_j V_j} + V_i \Sigma_k W_{ik} f_k^{\mu}\right]$$

terminal attractor
self-inhibition
recurrent excitation
gating inhibition
shunting interaction
thalamic input field



La dinamica è un rilassamento (poiché le connesioni laterali sono simmetriche) verso un punto di equilibrio modulato dal campo di eccitazione talamico.

Lo stato di equilibrio è un "population code" raccolto attorno alla microcolonna che riceve lo stimolo dominante.

La sharpness del population code è garantita da un meccanismo di "gating" inhibition (che bilancia la "recurrent excitation" senza usare una inibizione laterale) e da un meccanismo di "shunt" dell'ingresso talamico.

Speech motor control: articulatory & acoustic space

Field computation and speech control Cortical maps in the articulatory and acoustic maps

Training set: 5000 digitized X-ray images of the sagittal view of the vocal tract (the sampling frequency is $50Hz$) of a male french speaker, pronouncing VVV and VCV sequences (plosives and fricatives):

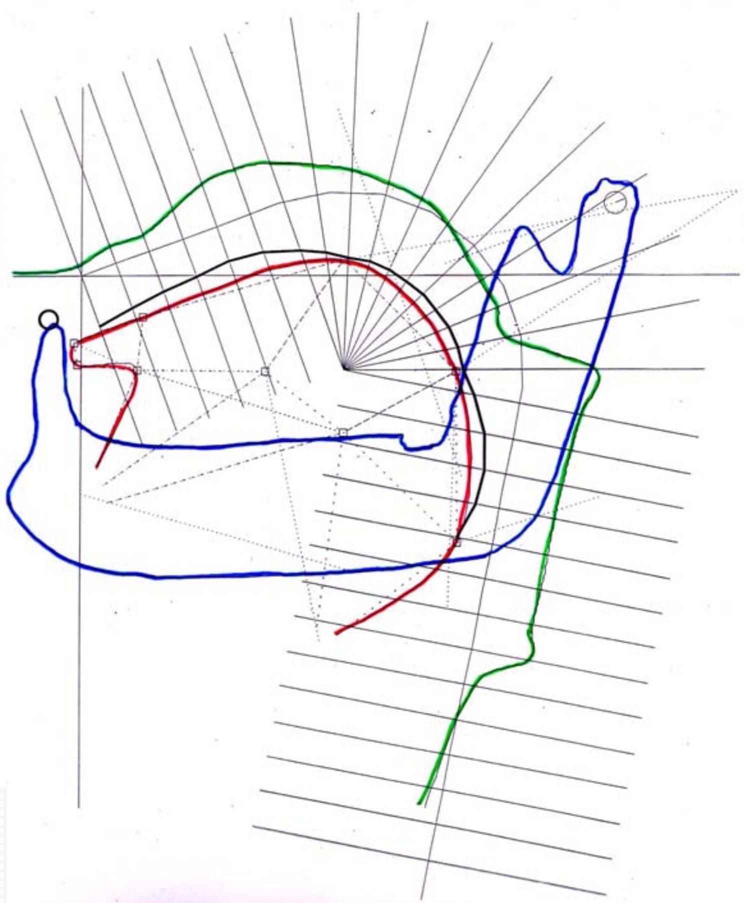
a ɛ e i y u o x
pava pavi pavu pivi pivu pivy
paza pazi pazu pizi pizu pizy
paZa paZi paZu piZi piZu piZy
aba abi abu ibi ibu iby
ada adi adu idi idu idy
aga agi agu igi igu igy

Embedding **acoustic space**: 5D

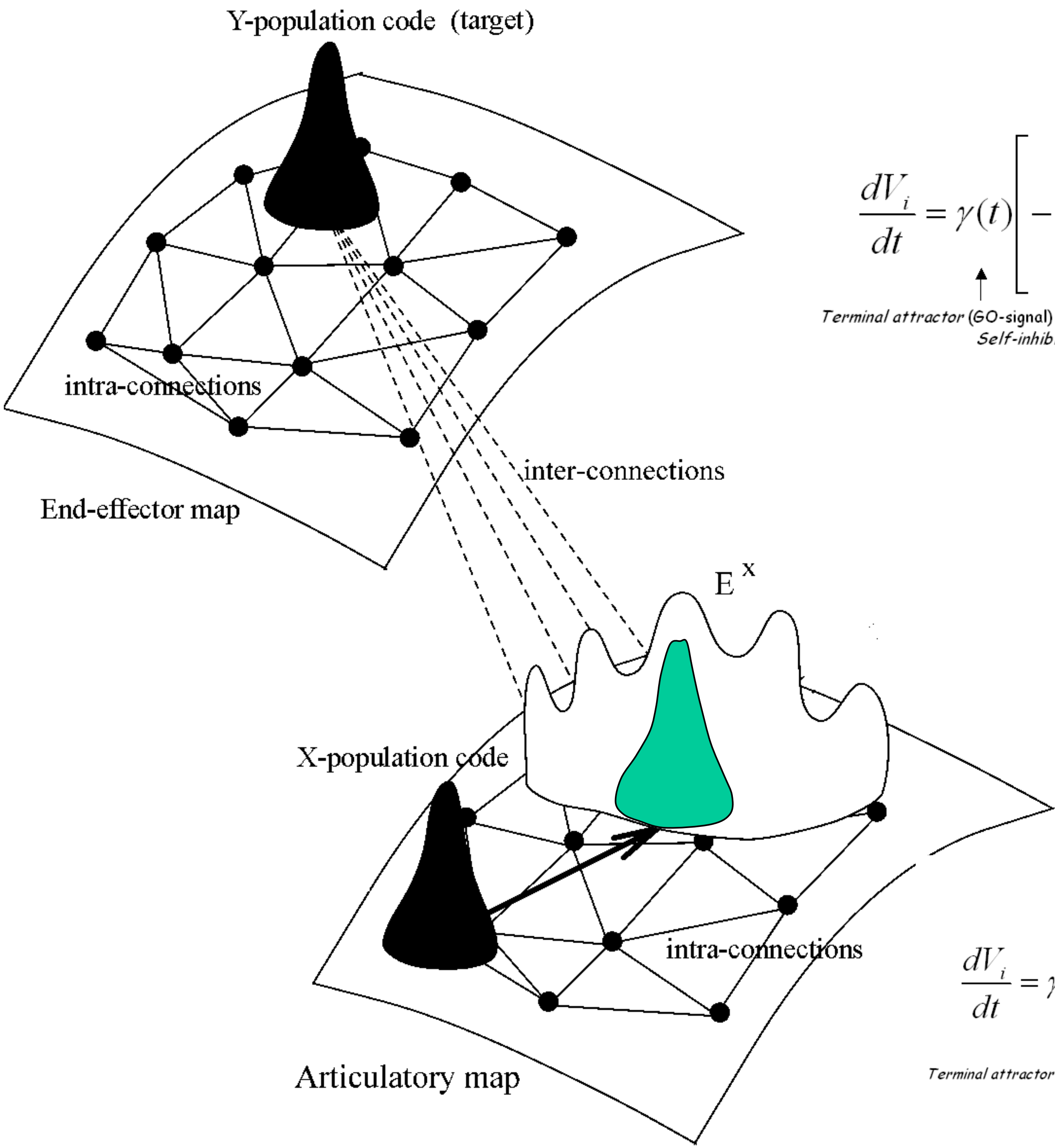
“Formants” (peaks in the spectrograms)

Embedding **articulatory space**: 10D

Parametrization of the vocal tract



LH – Lip Height
LP – Lip Protrusion
JH – Jaw Height
TB – Tongue Body
TD – Tongue Dorsum
TT – Tongue Tip
TA – Tongue Advance
LY – Larynx
VH – Velum Height
LV – Lips Vertical



Intra-connections

Inter-connections (Thalamo-cortical or cortico-cortical)

$$\frac{dV_i}{dt} = \gamma(t)\left[-\tau V_i + \sum_j C_{ij} \frac{V_j}{\sum_k V_k} + V_i \sum_r W_{ir} E_r \right]$$

Terminal attractor (GO-signal)

Self-inhibition

Recurrent excitation

Gating inhibition

Shunting interaction

Exexternal field

Intra-connections

Inter-connections (Thalamo-cortical or cortico-cortical)

$$\frac{dV_i}{dt} = \gamma(t)\left[-\tau V_i + \sum_j C_{ij} \frac{V_j}{\sum_k V_k} + V_i \sum_r W_{ir} E_r \right]$$

Terminal attractor (GO-signal)

Self-inhibition

Recurrent excitation

Gating inhibition

Shunting interaction

Exexternal field

Articulatory synthesis of the /*aεeiyuo*/ phonemic sequence.