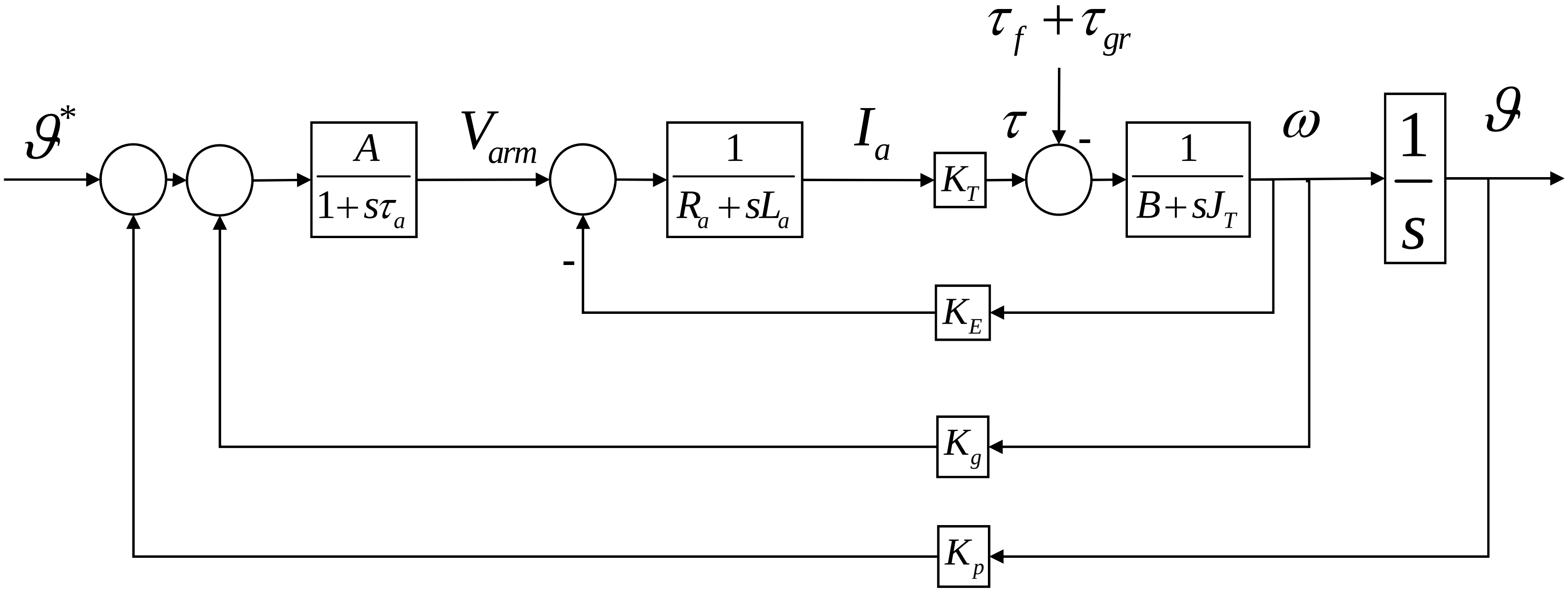
Overall…



$\tau_f + \tau_{gr}$
ϑ^*
$\frac{A}{1+s\tau_a}$
V_{arm}
$\frac{1}{R_a + sL_a}$
I_a
K_T
τ
-
$\frac{1}{B+sJ_T}$
ω
$\frac{1}{s}$
ϑ
-
K_E
K_g
K_p

Error and performance

$$\vartheta = \frac{\vartheta_d}{s} \qquad M(s) = \frac{K_T}{(R_a + sL_a)(B + sJ_T) + K_E K_T}$$

closed loop (velocity)

$$\omega(s) = \frac{\dfrac{A}{1+s\tau_a} M(s)}{1 + \dfrac{A}{1+s\tau_a} M(s) K_g}$$

$$\vartheta(s) = \frac{1}{s}\omega(s)$$

closed loop
(position)

$$\vartheta(s) = \frac{\dfrac{1}{s}\omega(s)}{1 + \dfrac{1}{s}\omega(s) K_p}$$

finally

$$\lim_{s\to 0} sH(s) = \lim_{t\to\infty} h(t)$$

$$\Rightarrow \lim_{s\to 0} s\frac{\vartheta_d}{s}\vartheta(s) = \lim_{s\to 0}\frac{s\frac{1}{s}\frac{\vartheta_d}{s}\omega(s)}{1+\frac{1}{s}\omega(s)K_p} = \frac{\vartheta_d}{K_p}$$

- For zero error K must be 1 or the control structure must be different

Same line of reasoning

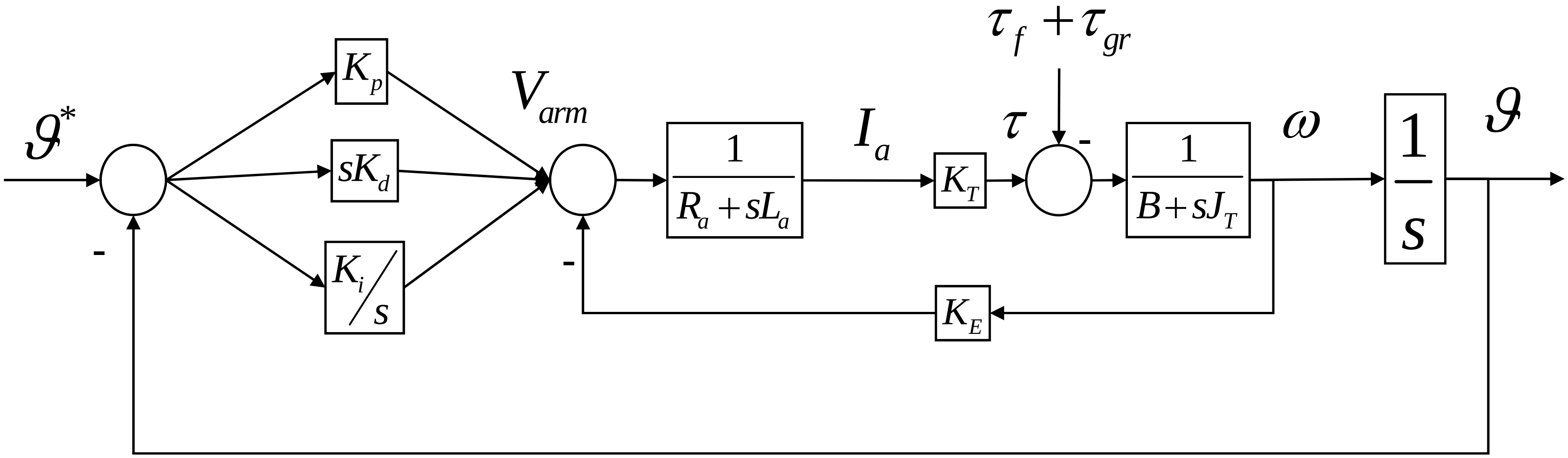
$$\vartheta_{final} = -\frac{\tau_{gr} R_a}{A K_T K_p}$$

- Final value due to friction and gravity

$$\left| \frac{\tau_{gr} R_a}{A K_T K_p} \right| \leq \vartheta_{\max} \Rightarrow K_p \geq \frac{\tau_{gr} R_a}{A K_T \vartheta_{\max}}$$

$$K_{p\min} = \frac{\tau_{gr} R_a}{A K_T \vartheta_{\max}}$$

PID controller



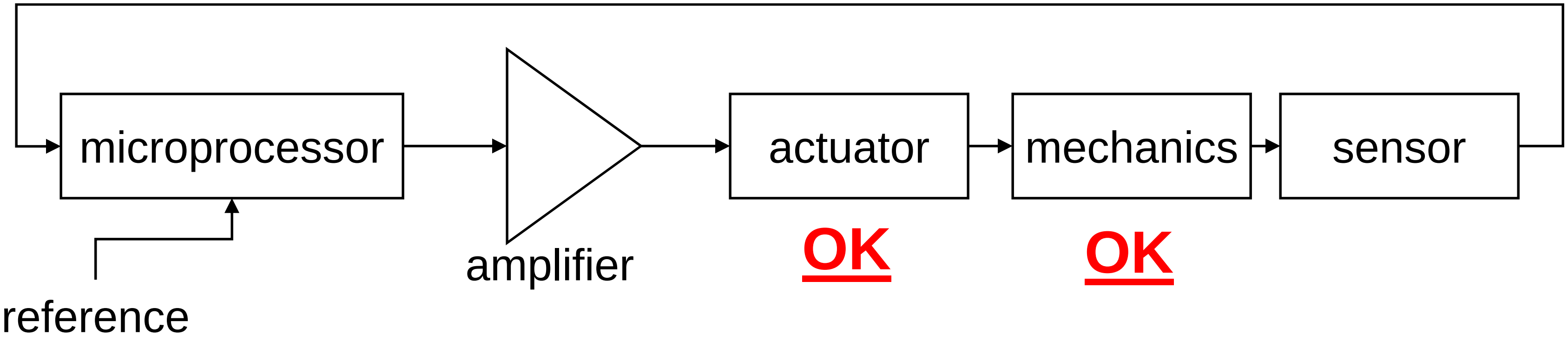
PID controller

- We now know why we need the proportional
- We also know why we need the derivative
- Finally, we add the integral
  - Integrates the error, in practice needs to be limited

Interpreting the PID

- Proportional: to go where required, linked to the steady-state error
- Derivative: damping
- Integral: to reduce the steady-state error

Global view



microprocessor

actuator

mechanics

sensor

amplifier

reference

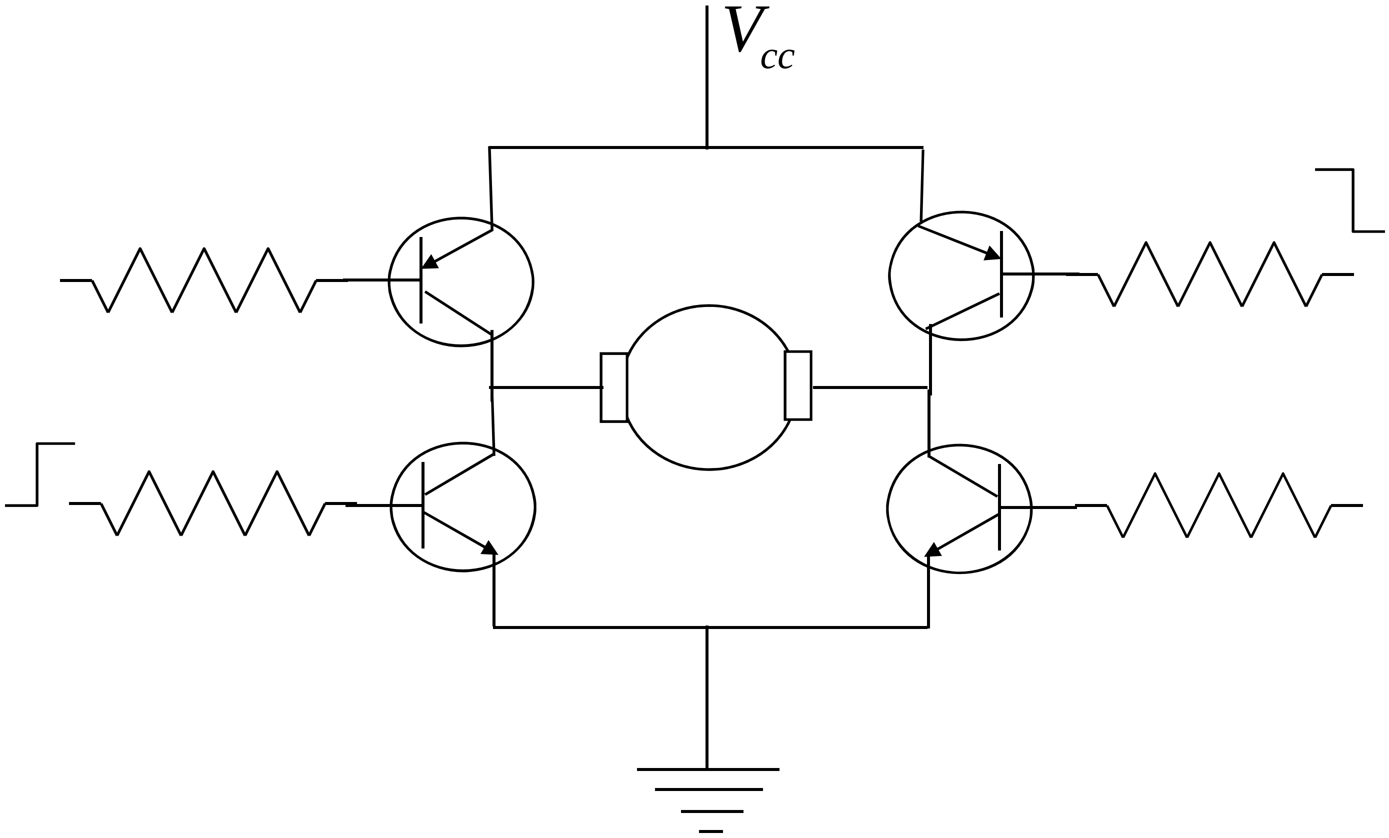
<u>OK</u>

<u>OK</u>

About the amplifiers

- Linear amplifiers
  - H type
  - T type
- PWM (switching) amplifiers

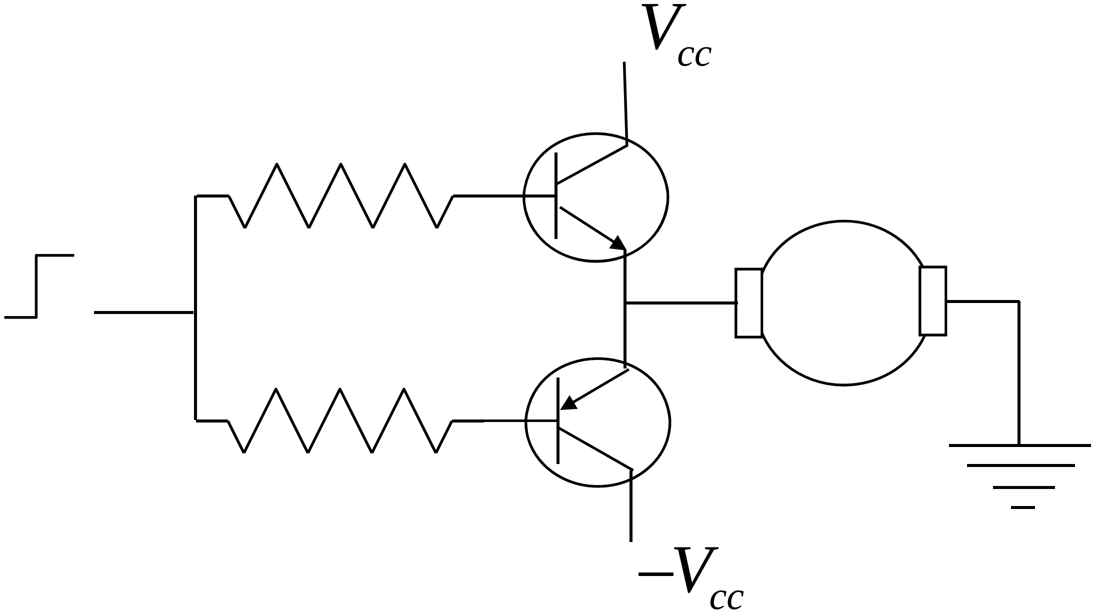
Let's consider the linear as a starting point



H-type

- The motor doesn’t have a reference to ground (floating)
- It’s difficult to get feedback signals (e.g. to measure the current flowing through the motor)

T-type



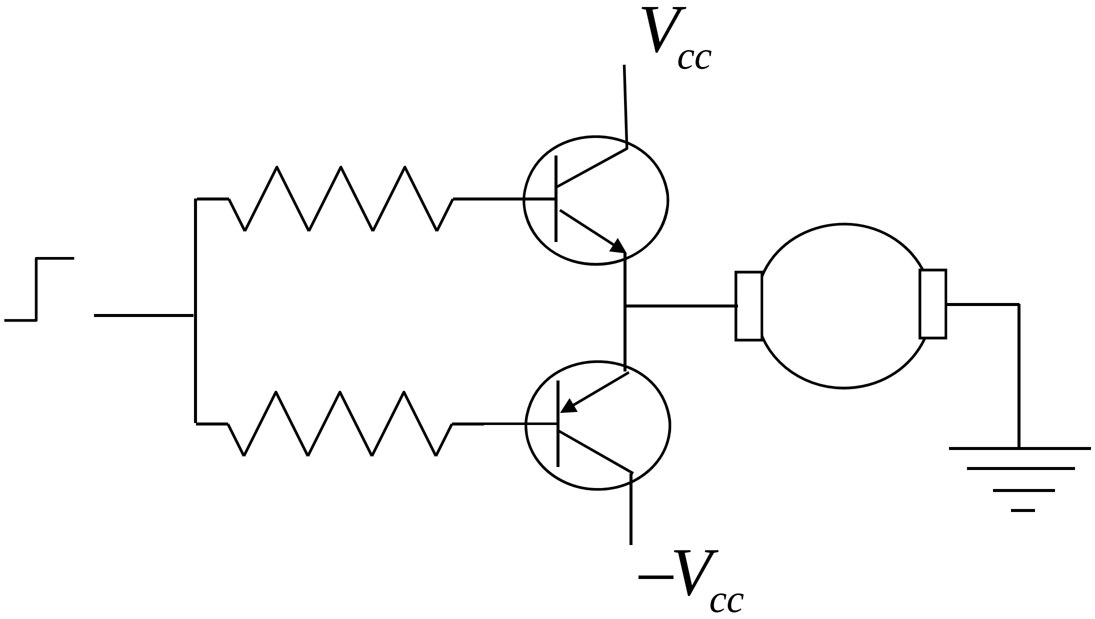
On the T-type

- Bipolar DC supply
- Dead band (around zero)
- Need to avoid simultaneous conduction (short circuit)

Things not shown

- Transistor protection (currents flowing back from the motor)
- Power dissipation and heat sink
  - Cooling
- Sudden stop due to obstacles
  - High currents → current limits and timeouts

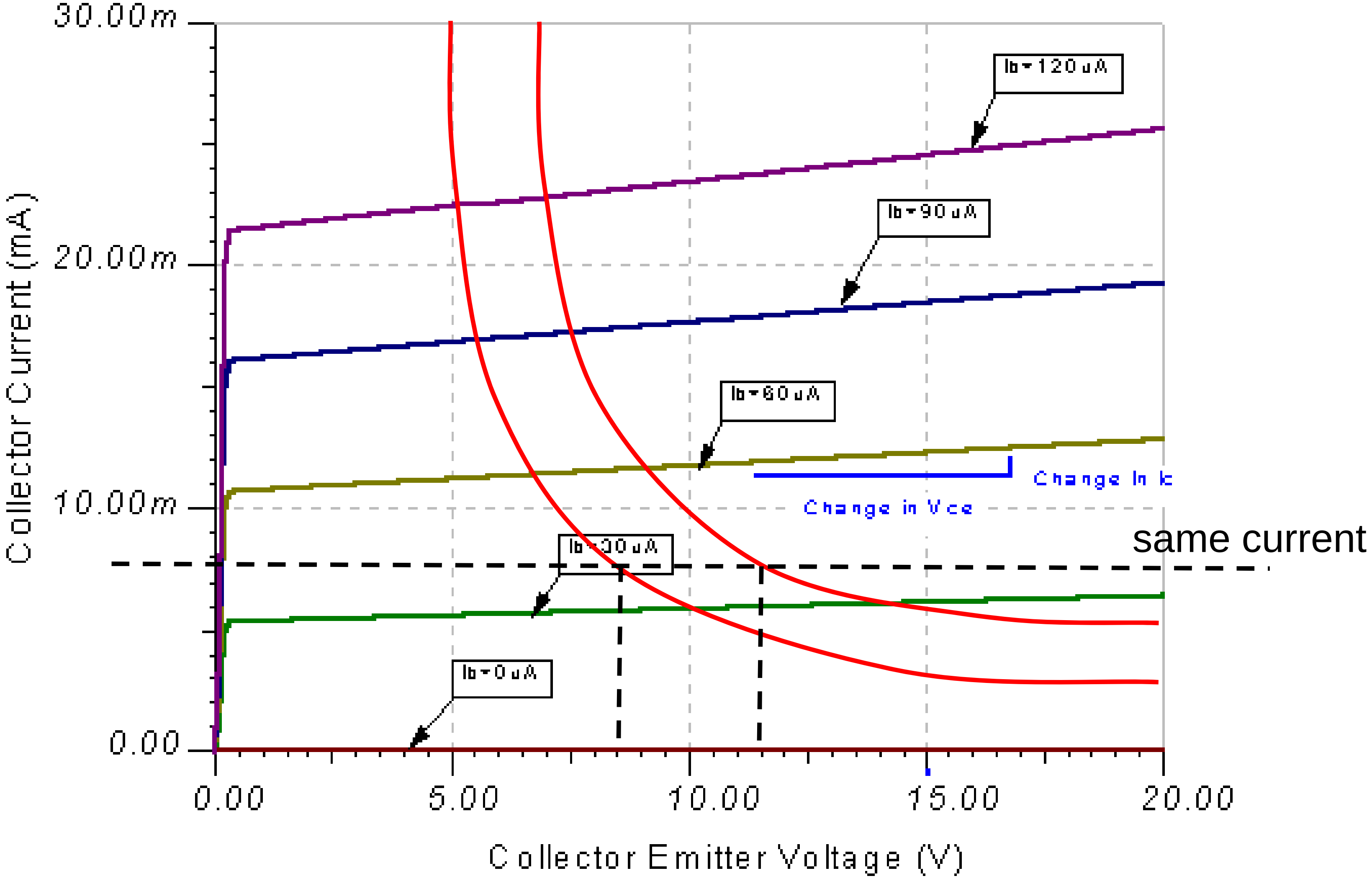
T-type



$$I_c \approx \frac{V_{cc}}{R_{transisor} + R_{motor}}$$

PWM amplifiers

$$P = V_{ce} I_c$$



PWM signal

$$P = V_{ce} I_c$$

- Transistors either “on” or “off”
  - When off, current is very low, little power too
  - When on, V is low, working point close to (or in) saturation, power dissipation is low

Comparison

- 12W for a 6A current using a switching amplifier
- 72W for a corresponding linear amplifier

Why does it work?

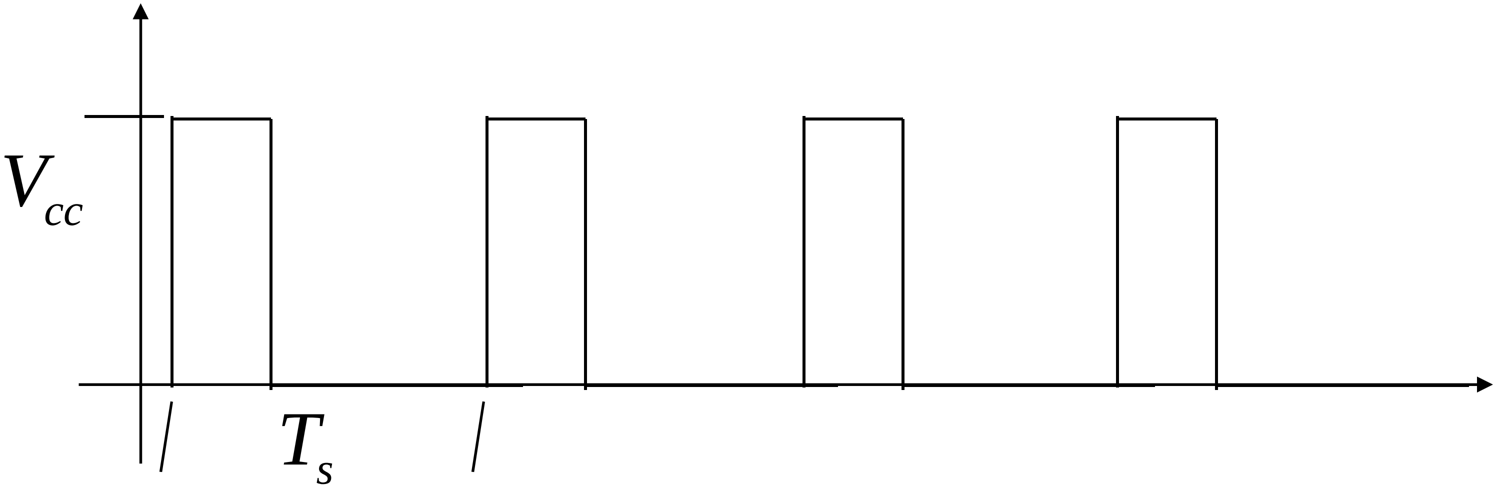
$$\frac{\omega(s)}{V_{arm}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B)/L_a J_T]s + (K_T K_E + R_a B)/L_a J_T}$$

- In practice the motor transfer function is a low-pass filter

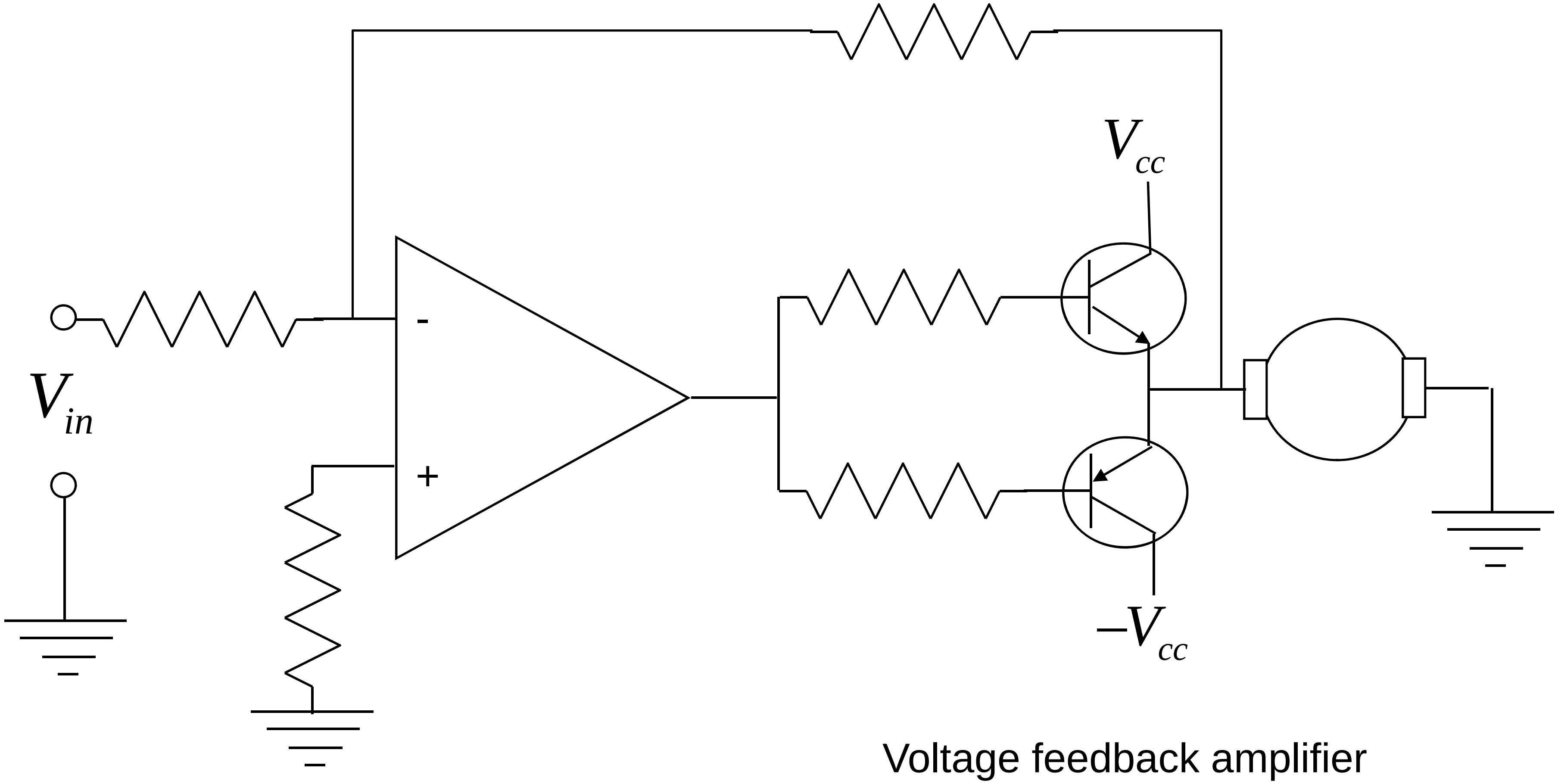
$$T_s \quad \text{with} \quad f_s \gg f_E (f_s > 100 f_E)$$

- Switching frequency must be high enough

PWM signal

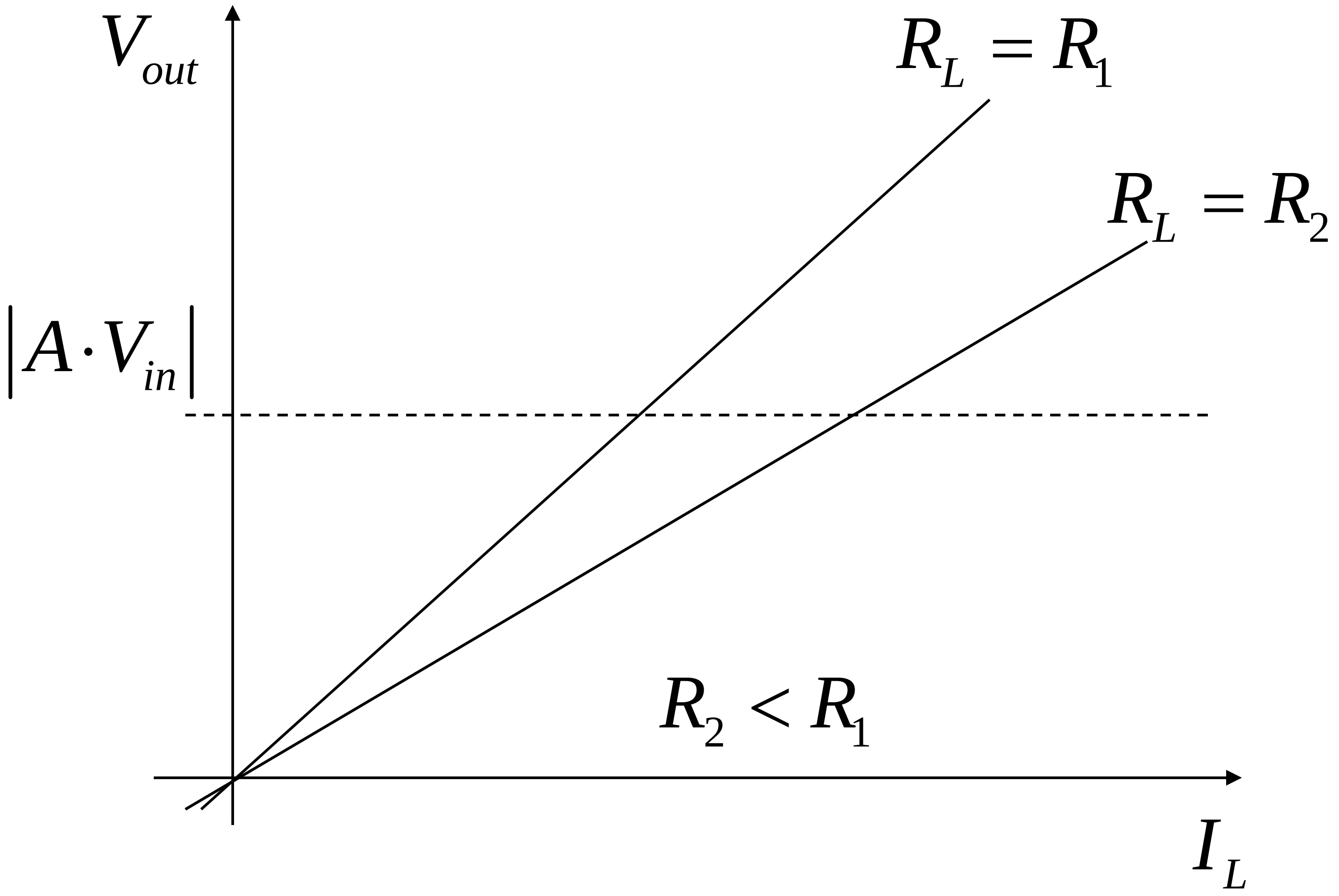


Feedback in servo amplifiers

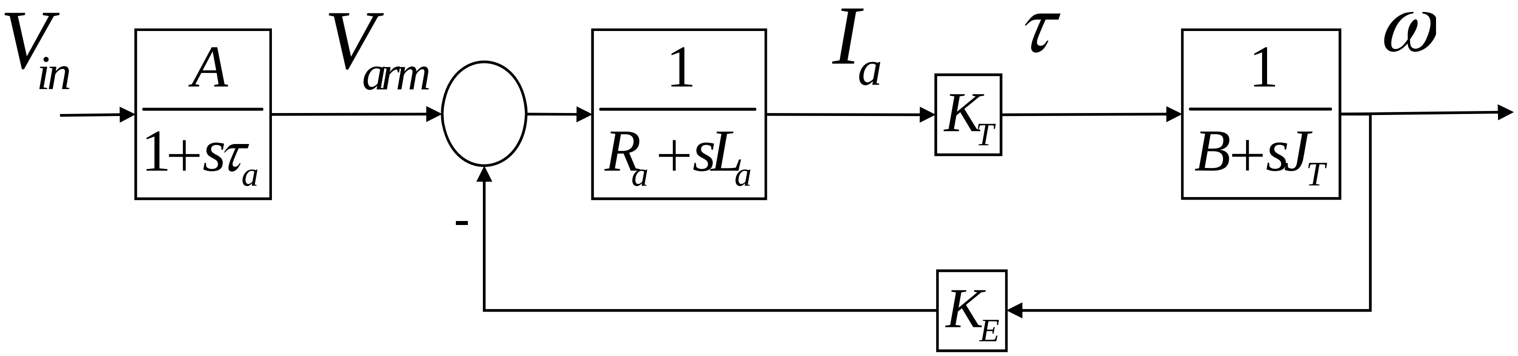


Voltage feedback amplifier

Operating characteristic

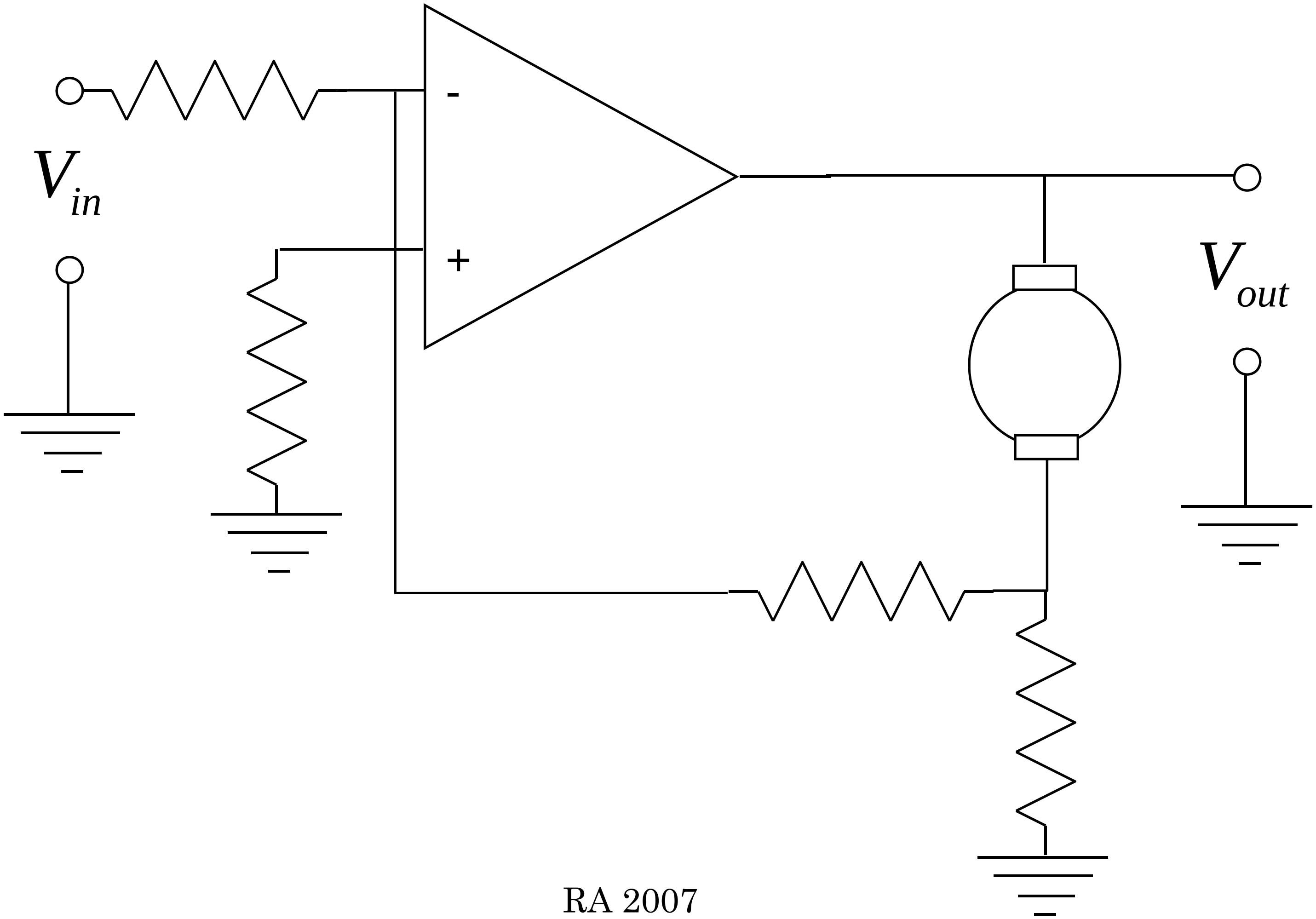


We've already seen this



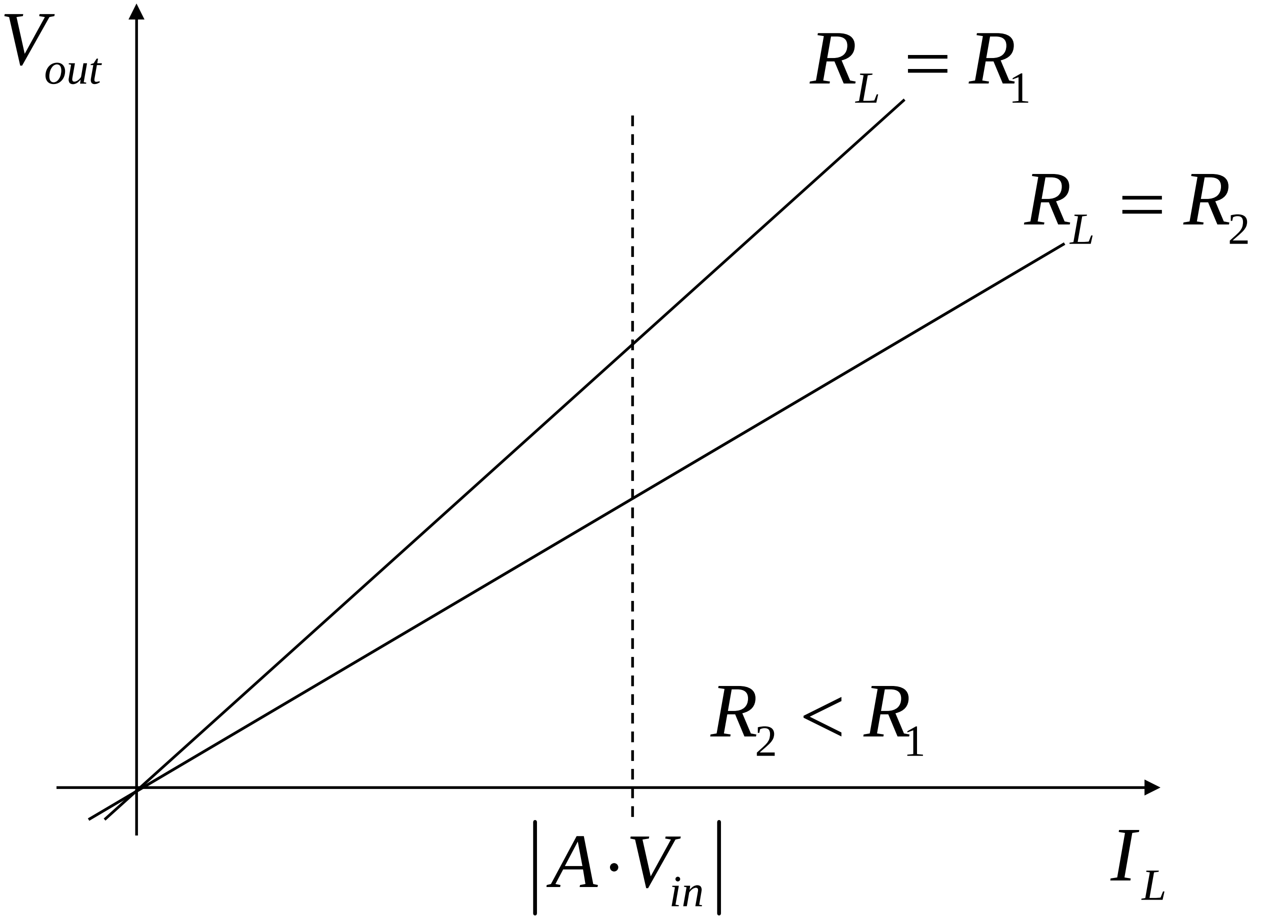
$$\frac{\omega(s)}{V_{in}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B)/L_a J_T]s + (K_T K_E + R_a B)/L_a J_T} \frac{A_v}{(1+s\tau_a)}$$

Current feedback

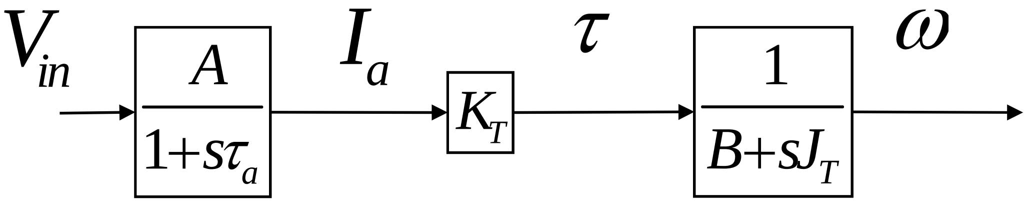


RA 2007

Current feedback



Motor driven by a current amplifier



$$\frac{\omega(s)}{V_{in}(s)} = \frac{K_T A_i}{(sJ_T + B)(1+s\tau_a)}$$

Bode plot analysis (in short)

$s = j\omega$ $\quad FT(j\omega)$ then plot $20\log\left|FT(j\omega)\right|$

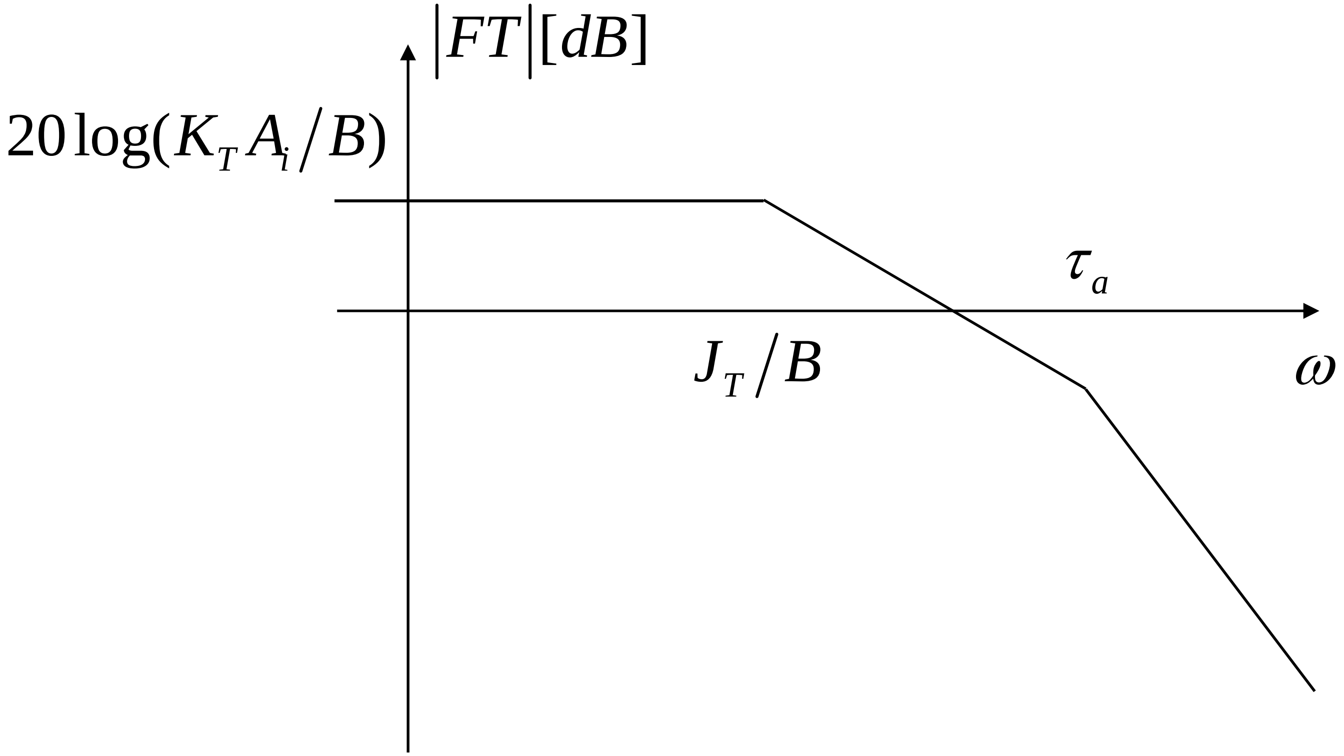
$\angle FT(j\omega)$

$$FT = K\frac{\Pi(1+{}^{j\omega}\!/\!_{\omega_{zi}})}{\Pi(1+{}^{j\omega}\!/\!_{\omega_{pk}})}$$

$$FT = 20\log K + 20\sum\log(1+\frac{\omega}{\omega_{zi}}) - 20\sum\log(1+\frac{\omega}{\omega_{pk}})$$

Example

$$\frac{\omega(s)}{V_{in}(s)} = \frac{K_T A_i / B}{(1 + s\,{}^{J_T}\!/_{B})(1 + s\tau_a)}$$



The (asymptotic) plot is accurate for…

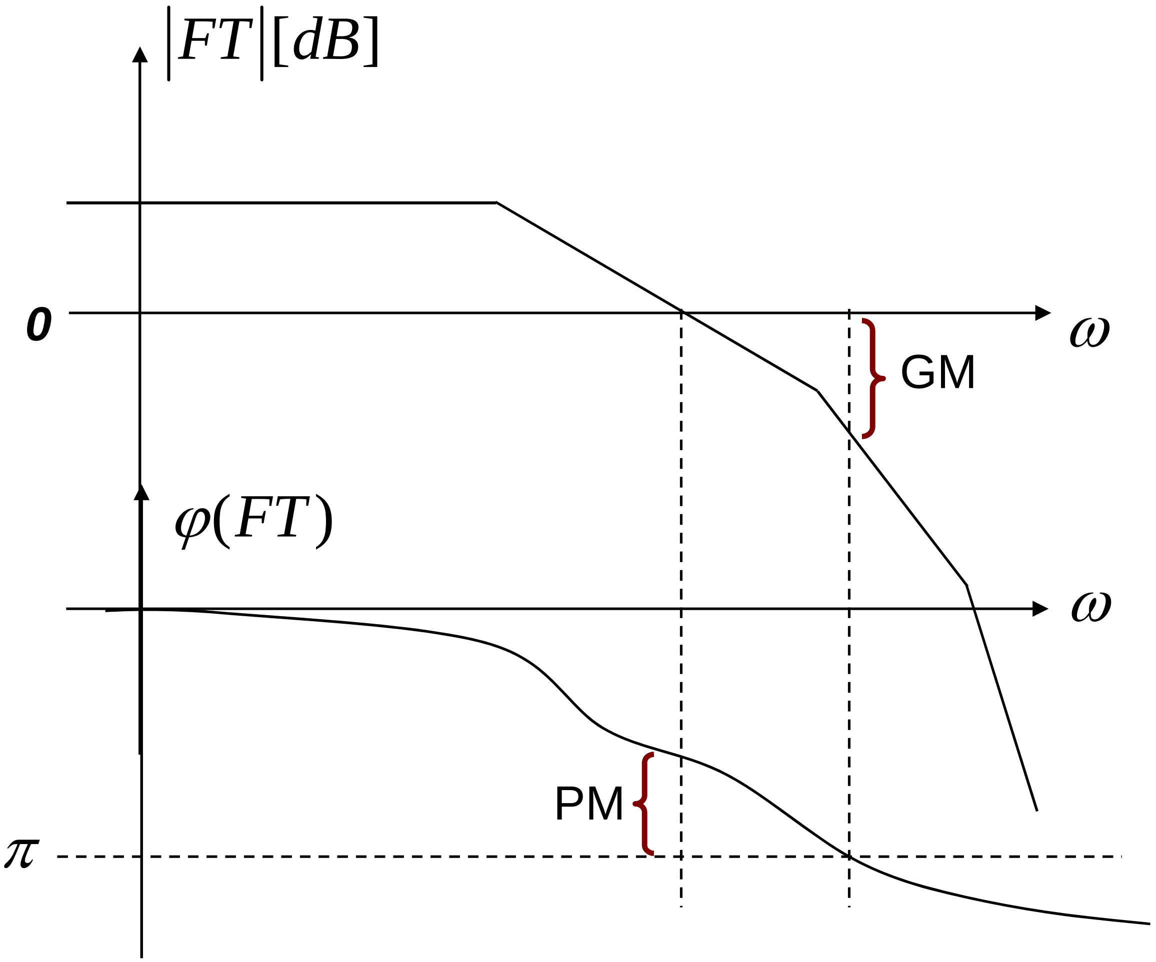
- Real valued poles and zeros, no resonance!
- Successive poles/zeros are separate by a factor of 7 or so, they don't interact

Gain and phase margin

$$GM = -20\log(|FT|) \quad @\,\omega_{\pi}$$

$$PM = \pi - \varphi(FT) \quad @\,\omega_{0}$$

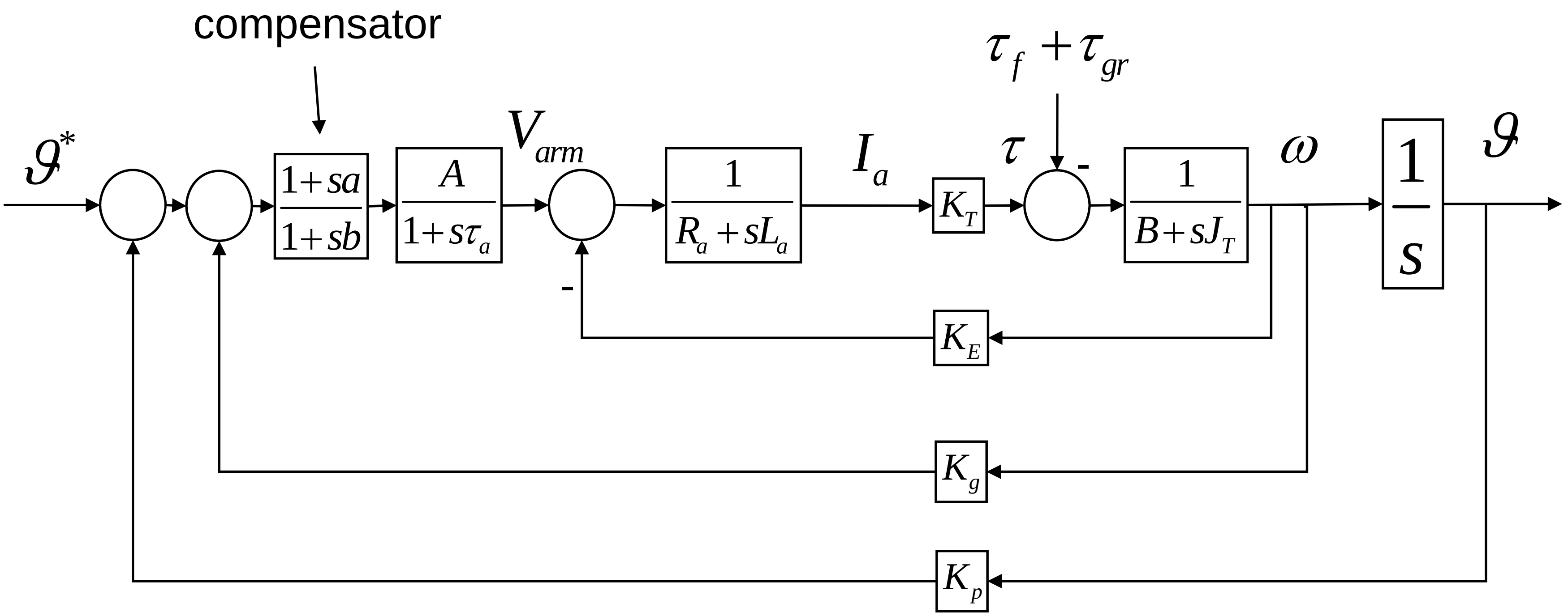
Margins



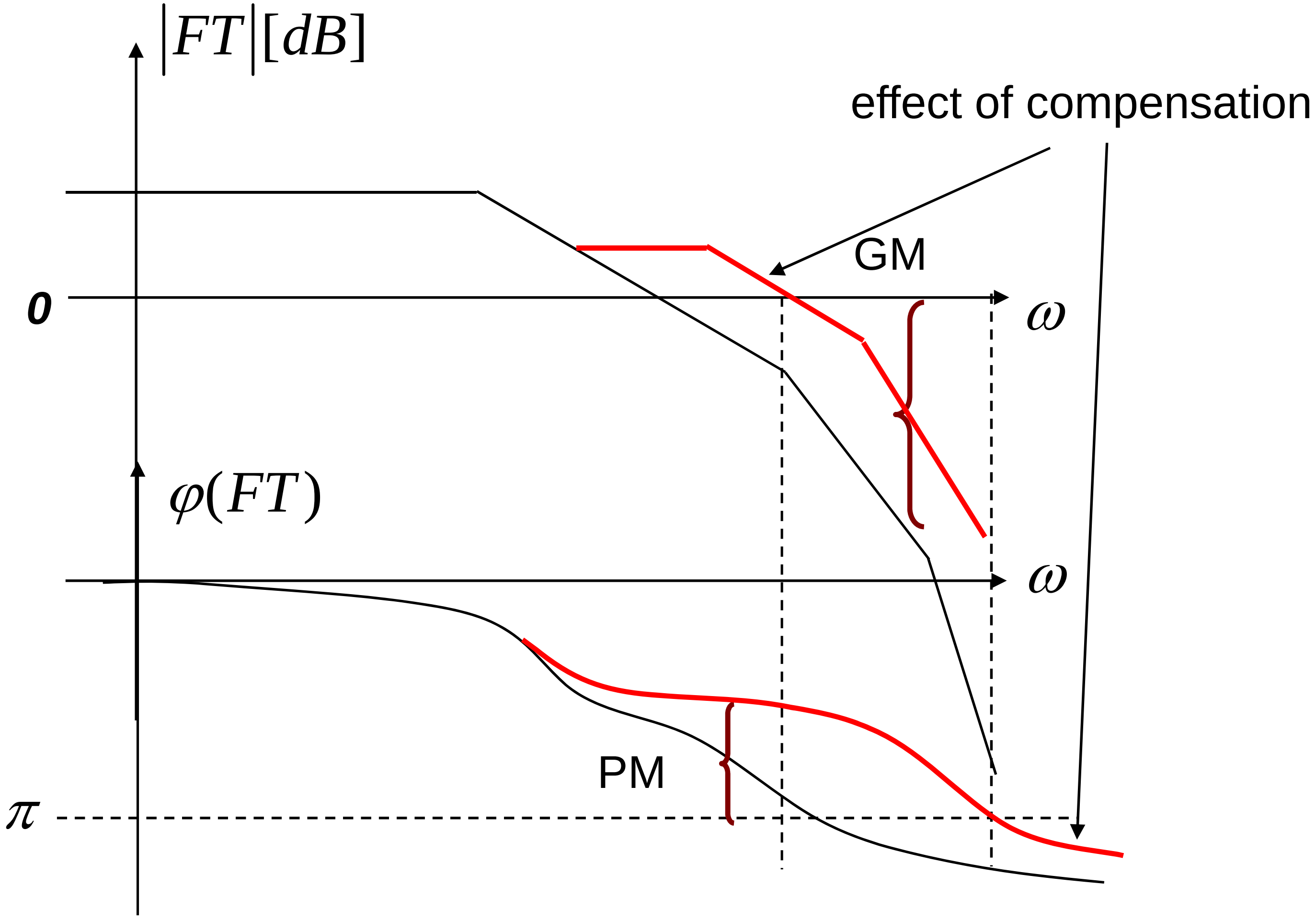
Rule of thumb

- Common design objectives:
  - Gain margin > 20dB
  - Phase margin > 45 degrees

Compensator

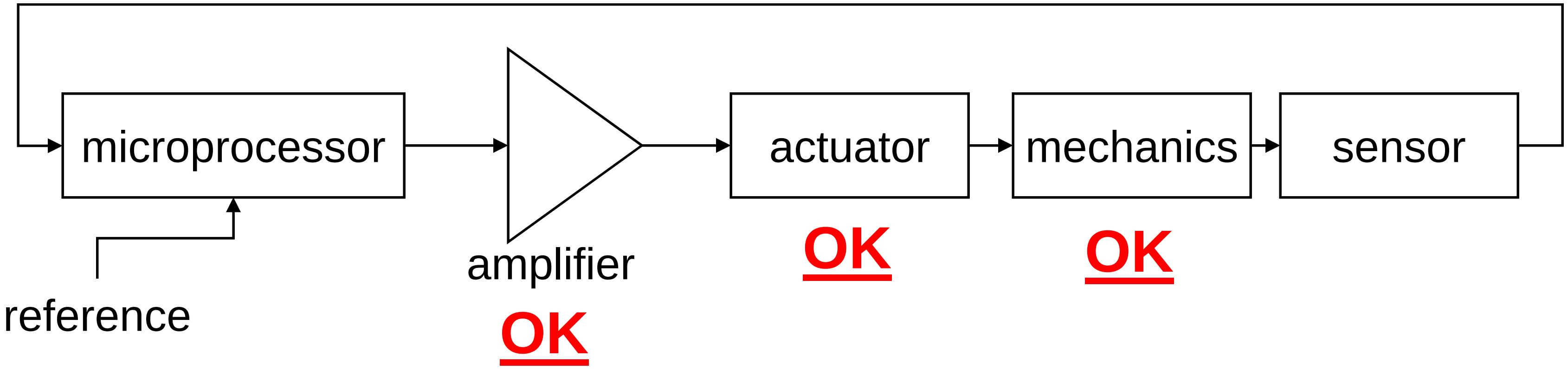


Effects of compensation



This plot is not a real one!

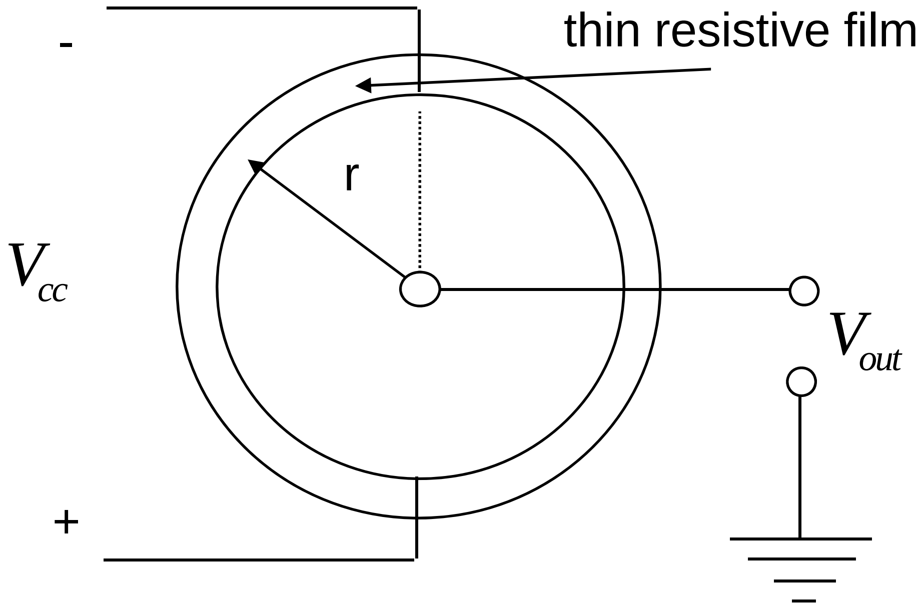
Back to the global view



Sensors

- Potentiometers
- Encoders
- Tachometers
- Inertial sensors
- Strain gauges
- Hall-effect sensors
- and many more…

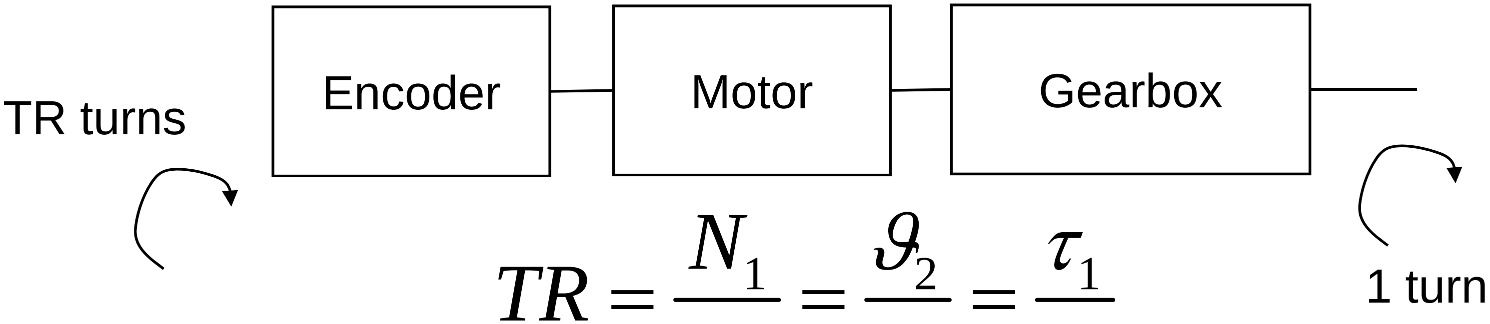
Potentiometer



$$V_{out} = \frac{r}{R} V_{cc}$$

- Simple but noisy
- Requires A/D conversion
- Absolute position (good!)

Note



$$TR = \frac{N_1}{N_2} = \frac{\vartheta_2}{\vartheta_1} = \frac{\tau_1}{\tau_2}$$

$$\tau_2 = \frac{N_2}{N_1}\tau_1 \Rightarrow \text{(most of the time)} N_2 > N_1$$

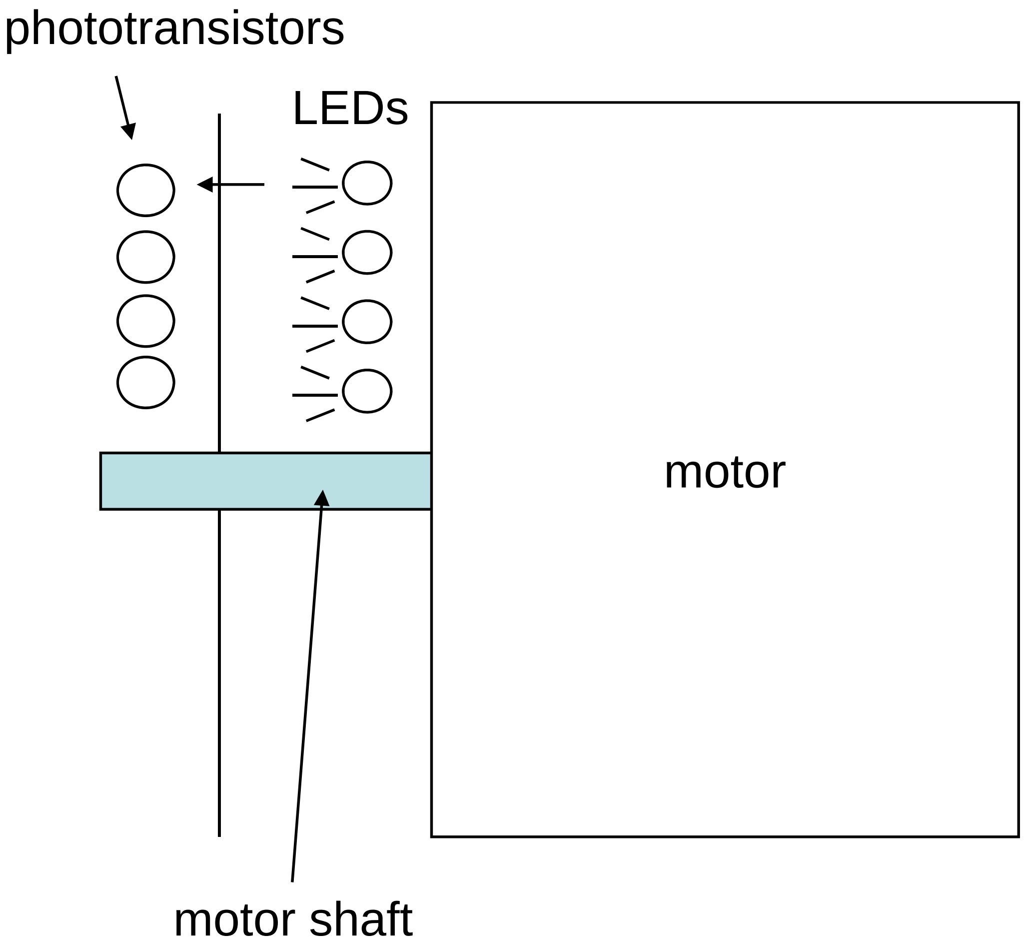
$$\vartheta_2 = \frac{N_1}{N_2}\vartheta_1$$

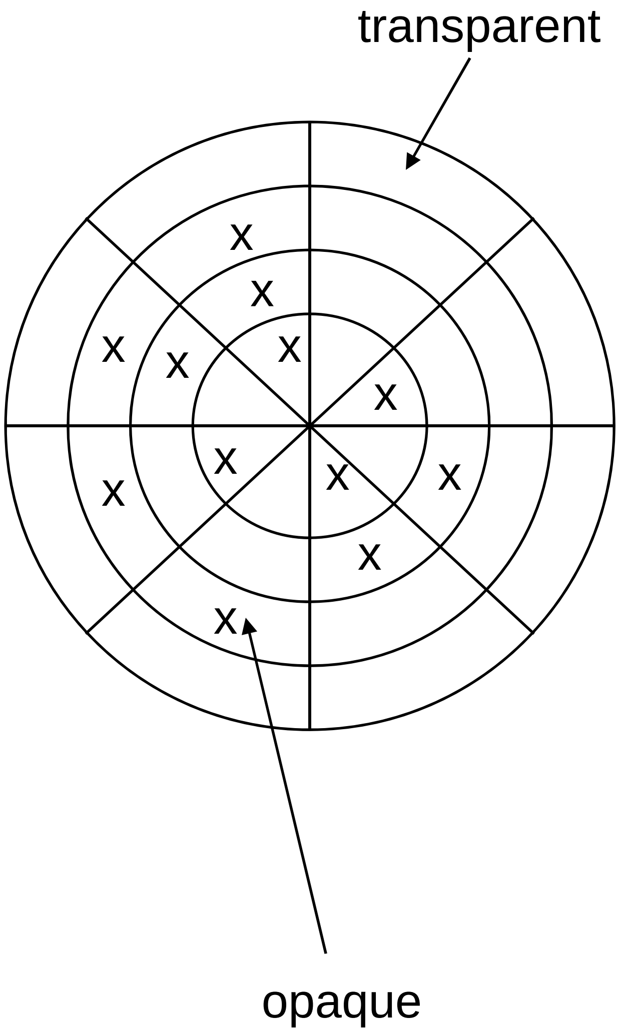
- The resolution of the sensor multiplied by TR

Encoder

- Absolute
- Incremental

Absolute encoder



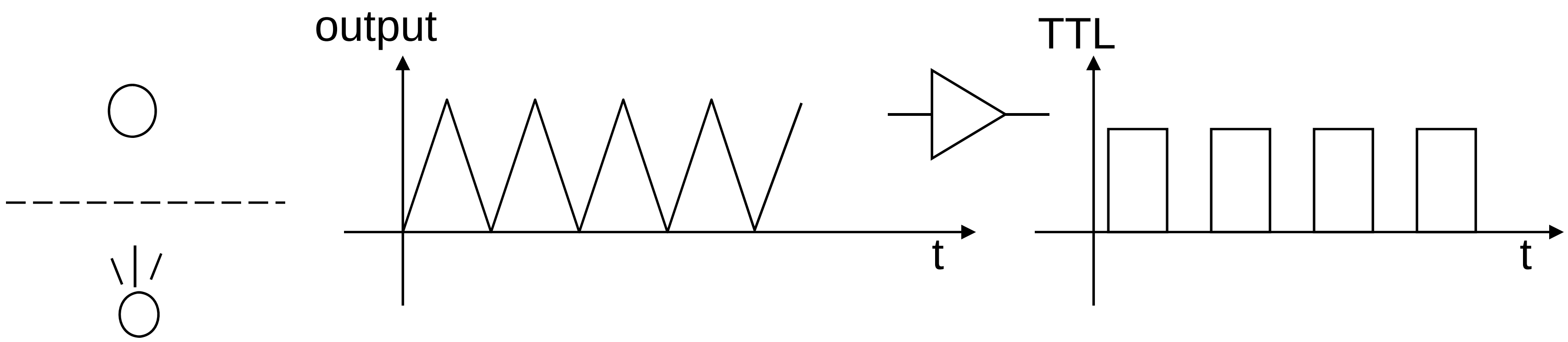


13 bits required for 0.044 degrees

Incremental encoder

- Disk single track instead of multiple
- No absolute position
- Usually an index marks the beginning of a turn

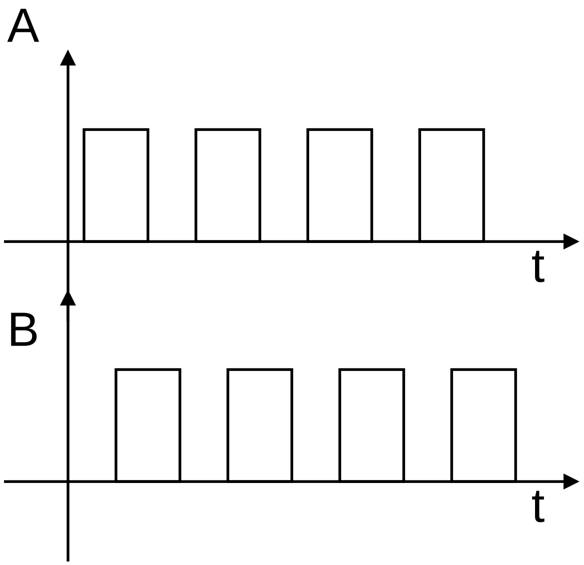
Incremental encoder

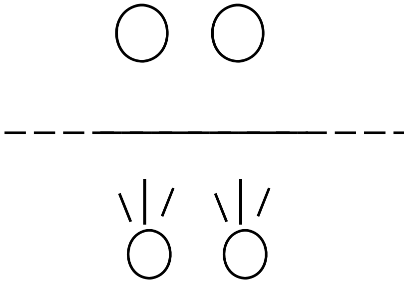


- Sensitive to the amount of light collected
- The direction of motion is not measured

Two-channel encoder

- 2 channels 90 degrees apart (quadrature signals) allow measuring the direction of motion





Moreover

- There are “differential” encoders
  - Taking the difference of two sensors 180 degrees apart
- Typically
  - A, B, Index channel
  - A, B, Index (differential)
- A “counter” is used to compute the position from an incremental encoder

Increasing resolution

- Counting UP and DOWN edges
  - X2 or X4 circuits

Absolute position

- A potentiometer and incremental encoder can be used simultaneously: the pot for the “absolute” reference, and the encoder because of good resolution and robustness to noise

Analog locking

- Use digital encoder as much as possible
  - Get to zero error or so using the digital signal
- When close to zeroing the error:
  - Switch to analog: use the analog signal coming from the photodetector (roughly sinusoidal/triangular)
  - Much higher resolution, precise positioning

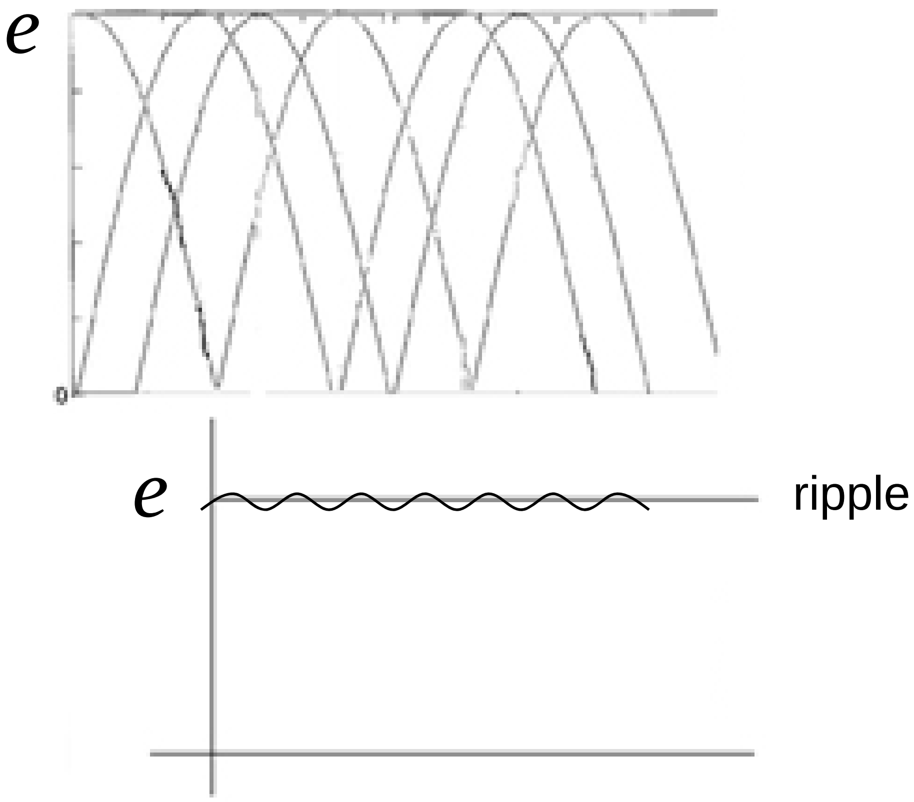
Tachometer

- Use a DC motor
  - The moving coils in the magnetic field will get an induced EMF

$$c\oint_{\delta s} \overline{E}\bullet d\overline{l} = \frac{d}{dt}\iint_{s} \overline{B}\bullet d\overline{S}$$

- In practice is better to design a special purpose “DC motor” for measuring velocity
- Ripple: typ. 3%

As already seen…



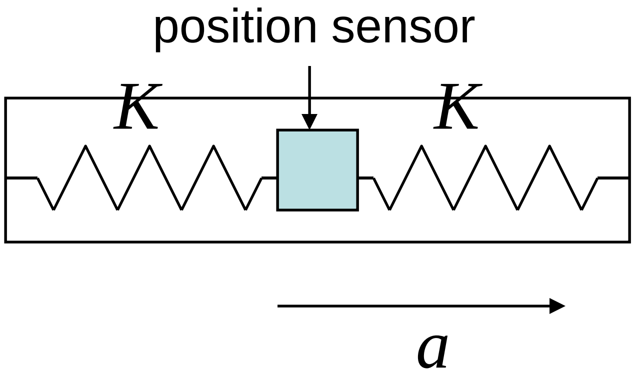
Measuring speed with digital encoders

- Frequency to voltage converters
  - – Costly (additional electronics)
- Much better: in software
  - – Take the derivative (for free!)

$$v(kT) = \frac{p(kT) - p((k-1)T)}{T}$$

Inertial sensors

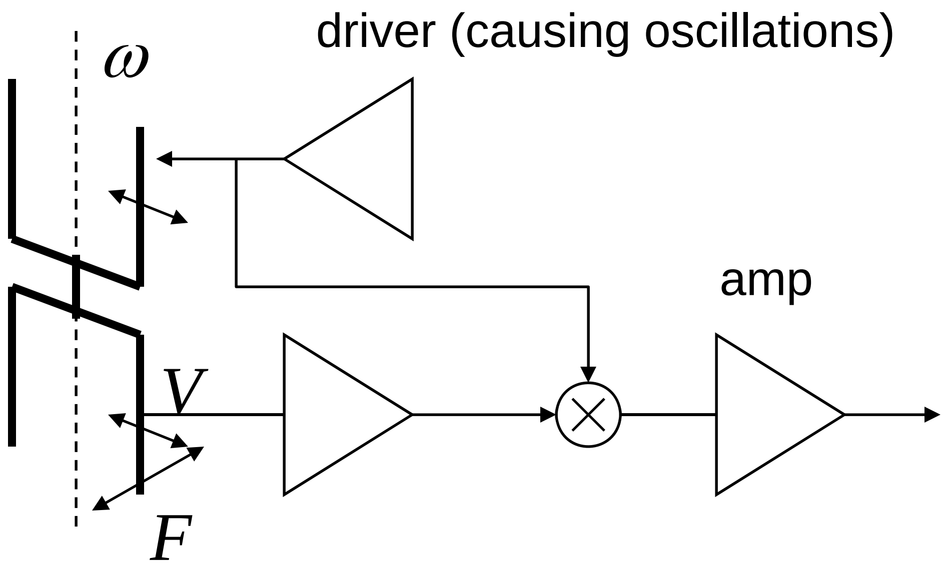
- Accelerometers:



$$Ma = 2Kx \Rightarrow a = \frac{2Kx}{M}$$

Gyroscopes

- Quartz forks



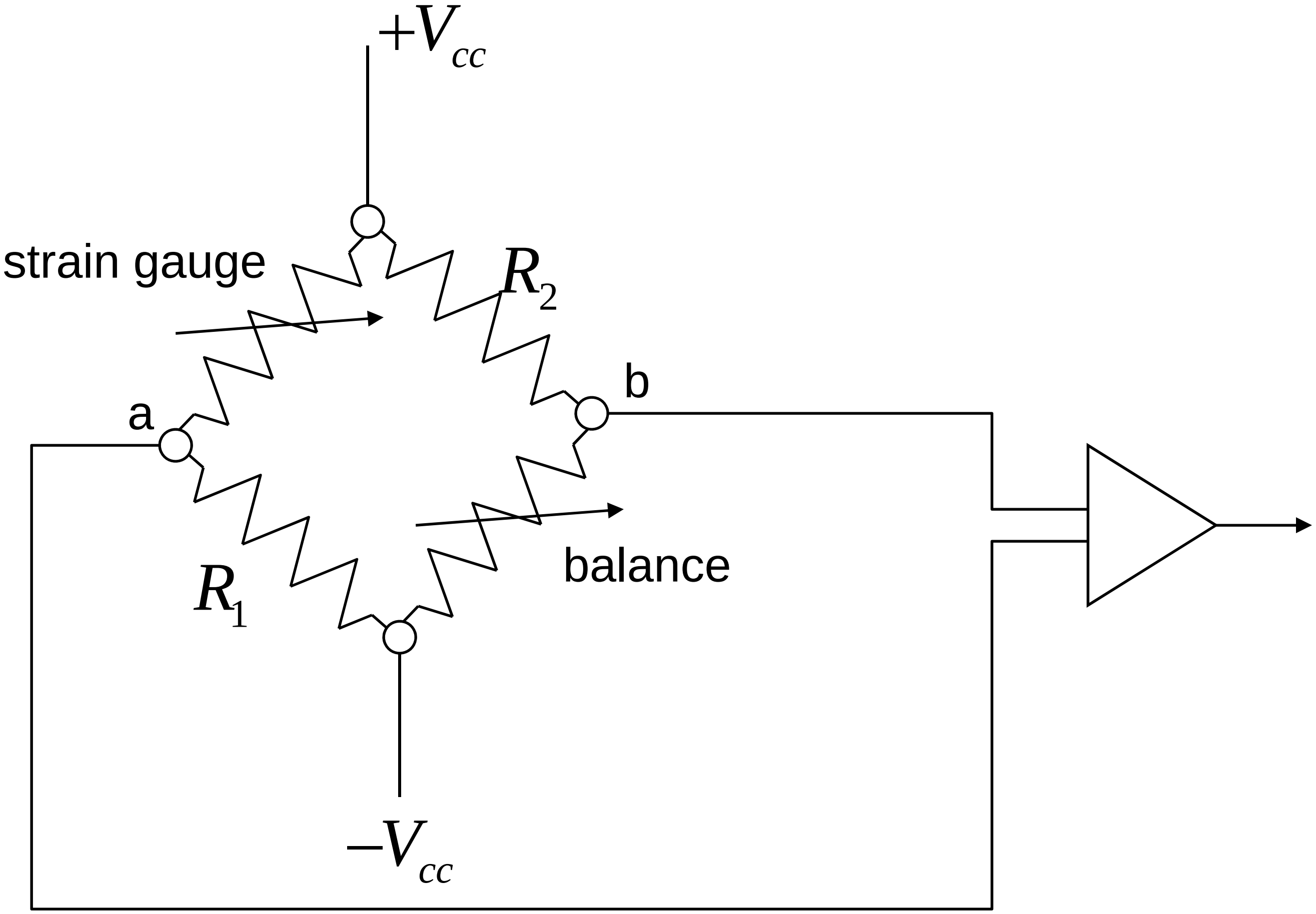
$$F = 2m\omega \times V$$

Strain gauges

- Principle: deformation →ΔR (resistance)
  - Example: conductive paint (Al, Cu)
  - The paint covers a deformable non-conducting substrate

$$R = \frac{L}{\underset{\substack{\uparrow \\ \text{conductivity}}}{\sigma} A} \Rightarrow \Delta L, A = const \Rightarrow \Delta R$$

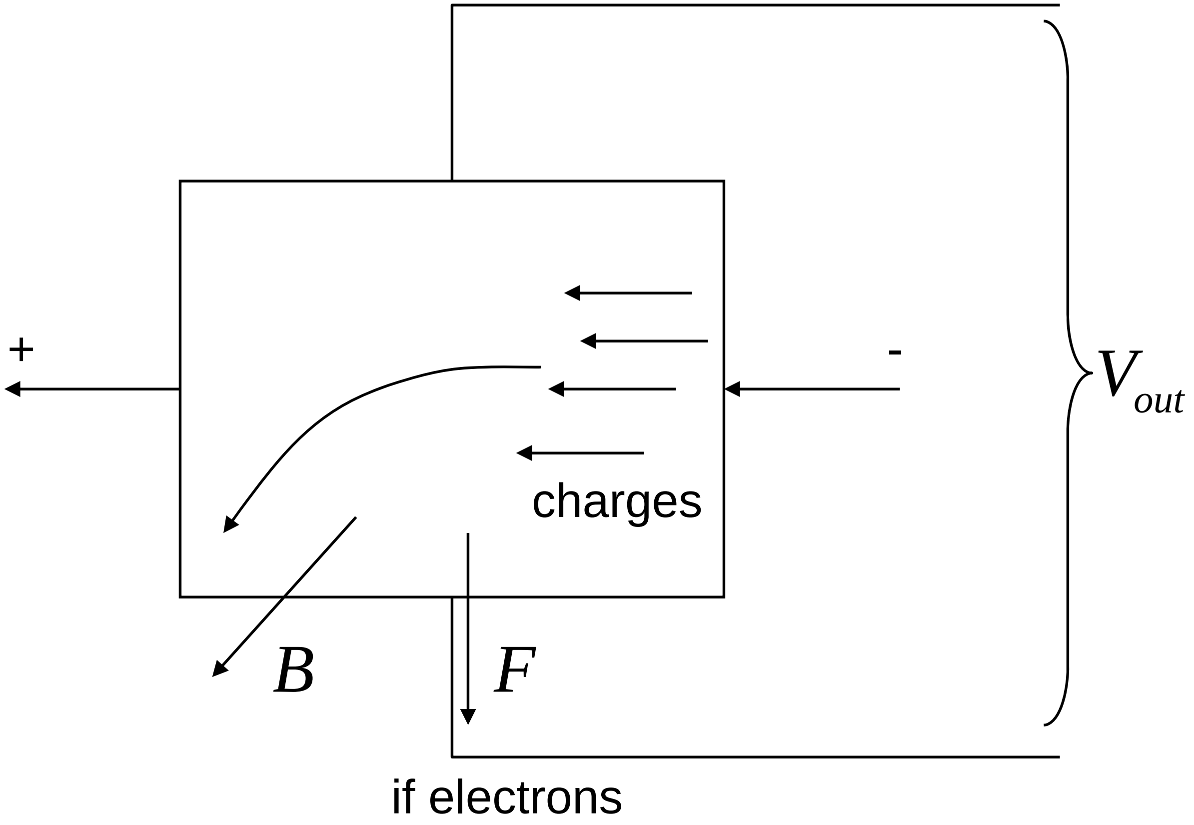
Reading from a strain gauge



$$R_1 R_2 = R_g R_b \Rightarrow V_{ab} = 0$$

$$\Delta V_{ab} = f(\Delta R_g)$$

Hall-effect sensors



$$F_{lorentz} = q\vec{v} \times \vec{B}$$

Example

- Measuring angles (magnetic encoders)

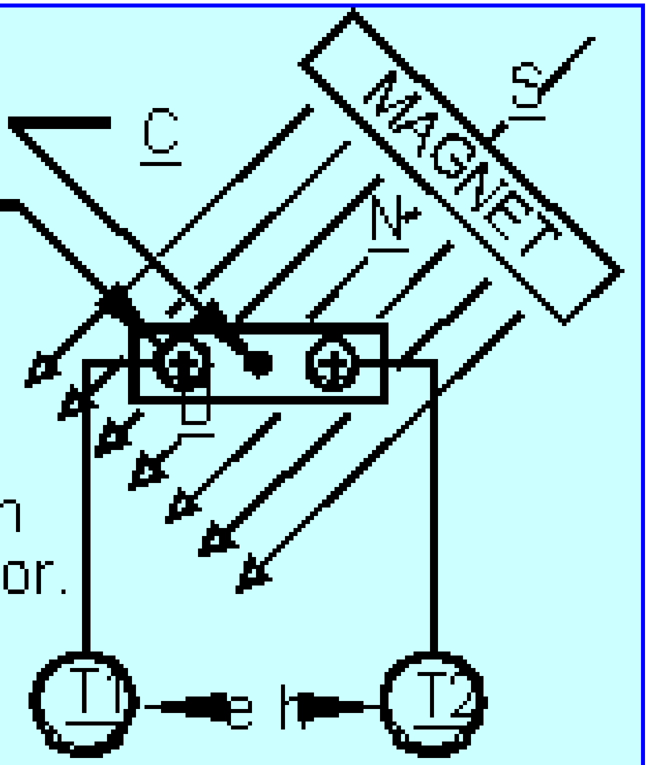
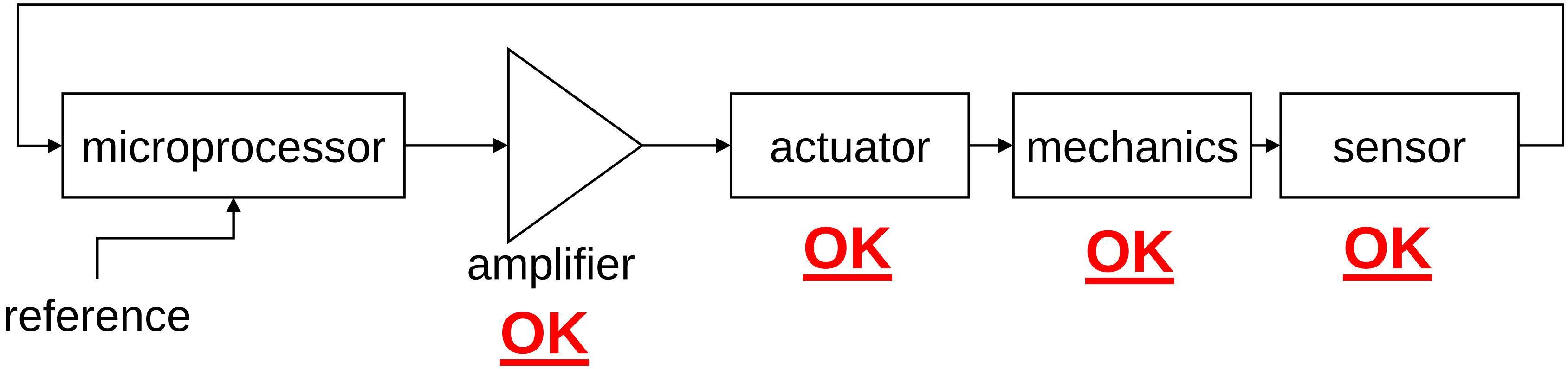


FIGURE 1--Basic Configuration of magnet and hall effect sensor. Edge views are shown.

Back to the global view



microprocessor

amplifier

actuator

mechanics

sensor

reference

OK

OK

OK

OK

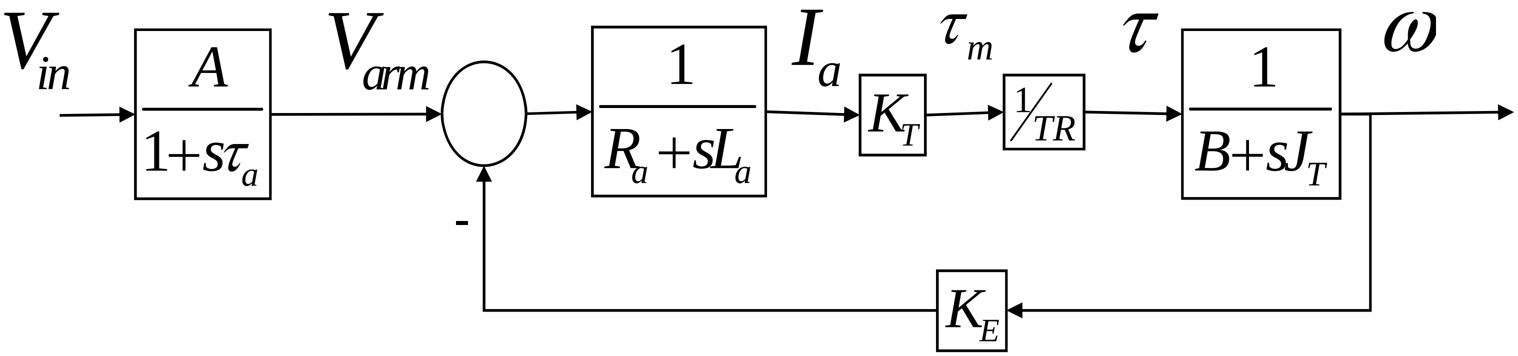
Microprocessors

- Special DSPs for motion control
  - Some are barely programmable (the control law is fixed)
  - Others are general purpose and they are mixed mode (analog and digital in a single chip)

Example

- DSP 16 bit ALU and instruction set
- PWM generator (simply attach this to either T or H amplifier)
- A/D conversion
- CAN bus, Serial ports, digital I/O
- Encoder counters
- Flash memory and RAM on-board
- Enough of all these to control two motors (either brush- or brushless)

Problem set



Simulate the following situation and build a controller for it.

- -B = 10*Bm
- -J = a thin bar 0.2m long and 0.2kg in weigth
- -Motor: 1331
- -A=1
- -ta=3ms
- -Add blocks as needed