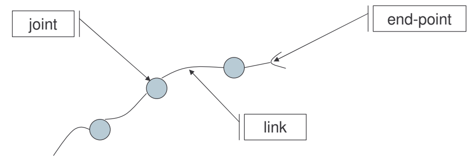
Robotica Antropomorfa

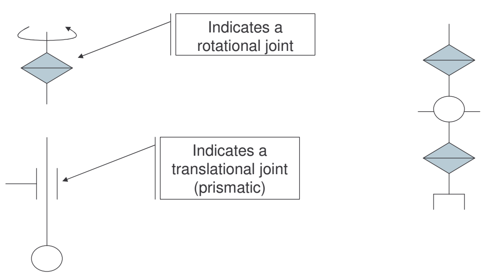
Lezione 2

Mechanical systems

- Things we'd like to model with some physics

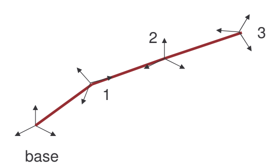


Sometimes in classical robotics



How to describe things mathematically

- One reference frame per link
  - Not needed for now…



Studying what?

	No forces	Forces
No motion	Styling	Static
Motion	Kinematics	Dynamics

Let's move to something simpler…

$F = \frac{d}{dt}(mv)$ Since links are physical objects with mass

$F = Mg$ Since in most cases gravity applies

$F = k(x - x_0)$ Springs

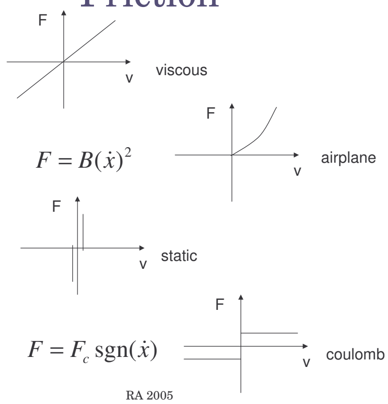
Friction

$F = B\dot{x}$

$F = B(\dot{x})^2$

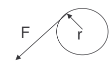
$F = \pm F_s\big|_{v=0}$

$F = F_c \operatorname{sgn}(\dot{x})$



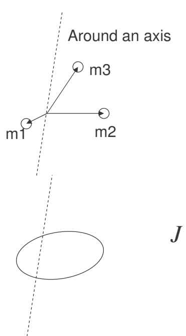
Rotational motion

$\tau = J\ddot{\vartheta}$ ***J = moment of inertia***



$\tau = F \times r$

Moment of inertia



$$J = \sum_{i=1}^{N} m_i r_i^2$$

density

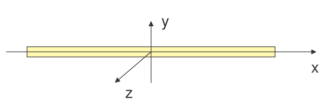
$$J = \int_{volume} \rho r^2 dV$$

Parallel axis theorem

$$J = J_c + Mr^2$$

Through the center of gravity

Example



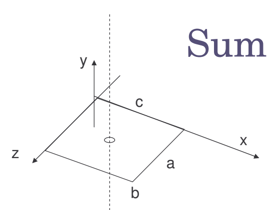
$Mass = M,\ \rho = M/l$

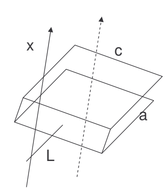
$J_x = 0$

$$J_y = \rho\int r^2 dV = \rho \int_{-l/2}^{l/2} x^2 dx = \rho \frac{1}{3} x^3\Big|_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

$$J_{y=-l/2} = \frac{Ml^2}{12} + M\frac{l^2}{4} = M\frac{l^2}{3}$$

Sum of *J*





$$J_x = \frac{M}{12}(a^2 + b^2)$$

$$J_y = \frac{M}{12}(a^2 + c^2)$$

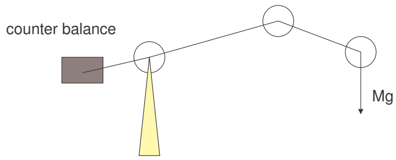
e.g. → $J_{top-x} = \frac{M_{top}}{12}(a^2 + c^2) + M_{top}(\frac{a}{2} + L)^2$

↓

$$J_z = \frac{M}{12}(b^2 + c^2)$$

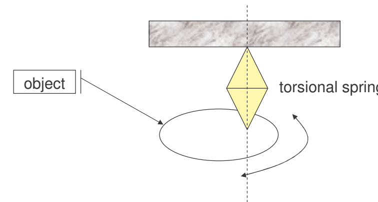
$J_{hand-x} = J_{top-x} + J_{side-x} + J_{bottom-x}$

Things you need to do for certain robots



- Balance a load against gravity (static)
- Higher moment of inertia (difficult to accelerate)

Experimental estimation of J

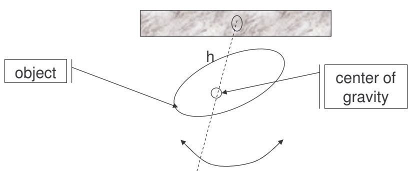


Use a photodiode and a computer to measure the frequency

Requires calibration from known J

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{J}}$$

Experimental estimation of J



$$f \approx \frac{1}{2\pi}\sqrt{\frac{Mgh}{J}}$$

Work and power

- Needed when talking about dynamics

$E = const$ **if** $\sum F_{ext} = 0$

$W = \int_{s1}^{s2} F ds$ $\quad W = \Delta E, E = \text{energy}$

$W = \frac{1}{2}Mv_1^2 - \frac{1}{2}Mv_2^2$ work done to change the kinetic energy

$W = Mgh_1 - Mgh_0$ work done to move from 0 to 1 (against the gravity field)

$P = \frac{dW}{dt}$ Power → $P = Fv$

Power is sometimes dissipated in heat

$$P = Fv \quad P_{friction} = Bv^2$$

Rotational case

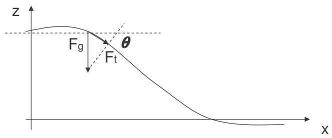
$E = const$ **if** $\sum \tau_{ext} = 0$

$W = \int_{\vartheta 1}^{\vartheta 2} \tau d\vartheta$ $\quad W = \Delta E, E = \text{energy}$

$K = \frac{1}{2}J\omega^2$ **kinetic energy**

$P = \frac{dW}{dt}$ Power → $P = \tau\omega$

A sort of example



$$\frac{1}{2}mv^2 + mgh = const \qquad mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0$$

$$mv\frac{dv}{dt} = mav \Rightarrow F_t v \Rightarrow -F_g \sin\vartheta\, v \Rightarrow -mg\sin\vartheta\, v$$

$NB: v$ is the tangential velocity

$$\Rightarrow -mg(\sin\vartheta v) = -mg\frac{dh}{dt}$$

RA 2005