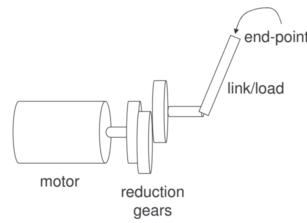
Robotica Antropomorfa

Lezione 3

OS 2005

Continuing the modeling of the single joint



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Motor

- Let's imagine for now that it is something that generates a given torque

RA 2005

Mechanical transmission

- Gears
- Belts
- Lead screws
- Cables
- Cams
- etc.

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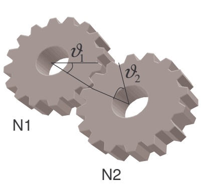
Gears



- Distance traveled is the same:

$$r_1 \vartheta_1 = r_2 \vartheta_2$$

- Because the size of teeth is the same:

$$\frac{N_1}{r_1} = \frac{N_2}{r_2}$$

RA 2005

Gearhead (for real)

Furthermore…

$$r_1\vartheta_1 = r_2\vartheta_2$$

$$\frac{N_1}{r_1} = \frac{N_2}{r_2}$$

- No loss of energy $\tau_1\vartheta_1 = \tau_2\vartheta_2$

Combining…

$$\frac{N_1}{N_2} = \frac{r_1}{r_2} = \frac{\vartheta_2}{\vartheta_1} = \frac{\tau_1}{\tau_2} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

# of teeth

Inverse relationship between speed and torque

$$\tau_2 = \tau_1 \frac{N_2}{N_1} \qquad TR = \frac{N_1}{N_2}$$

input

output

mechanical parameter

Equivalent J

$$\ddot{\vartheta}_1 J_1 \Leftarrow \tau_1 = \tau_2 \frac{N_1}{N_2} = \ddot{\vartheta}_2 J_2 \frac{N_1}{N_2}$$

$$J_1 = \frac{\ddot{\vartheta}_2}{\ddot{\vartheta}_1} J_2 \frac{N_1}{N_2} \Rightarrow \left(\frac{N_1}{N_2}\right)^2 J_2$$

$$J_1 = TR^2 J_2$$

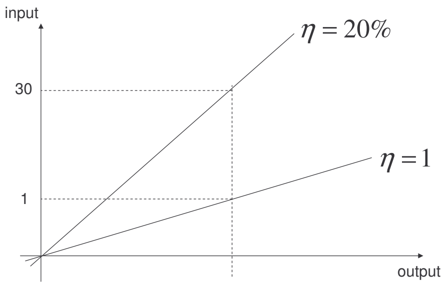
- J as seen from the motor

In reality

$$\tau_{out} = \tau_{in} \frac{1}{TR} \eta$$

- Where η is the efficiency of the mechanism
- η is related to power, speed ratio doesn't change
- η is also the ratio of input power vs. power at the output

For example



Example

- From catalog (reduction gearbox):
  - 4:1 =90%
  - 16:1 =80%
  - 64:1 =70%
  - 256:1 =60%
  - 1024:1 =55%

Motion conversion

- Start with

$$\tau_2 = \frac{N_2}{N_1}\tau_1$$

- Design *TR*, more torque (usually)

$$TR < 1$$

$$N_2 > N_1$$

$$J_1 < J_2 \Leftrightarrow \omega_2 < \omega_1$$

Viscous friction

- Easy:

$$\tau_{viscous} = B_2\dot{\vartheta}_2$$

$$\tau_{eq\_viscous} = TR \cdot \tau_{viscous} = TR \cdot B_2\dot{\vartheta}_2$$

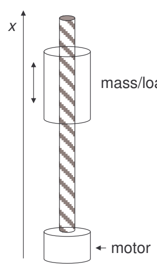
$$B_{eq}\dot{\vartheta}_1 = TR \cdot B_2\dot{\vartheta}_2 \Rightarrow B_{eq} = TR^2 B_2$$

- Coulomb friction:

$$\tau_{eq} = TR \cdot F_c \,\mathrm{sgn}(\dot{\vartheta}_2)$$

Lead screw

- Rotary to linear motion conversion



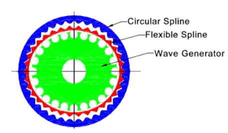
$$\vartheta[rad] = 2\pi P x$$

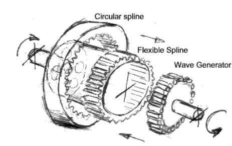
$$\dot{\vartheta} = 2\pi P \dot{x}$$

$$E_{rot} = E_{lin} \Rightarrow \frac{1}{2}M_{load}v^2 = \frac{1}{2}J\omega^2 \Rightarrow$$

$$\Rightarrow J = \frac{M_{load}}{(2\pi P)^2}$$

Harmonic drives





From the harmonic drive website
http://www.harmonicdrive.de

Example

- Designing the single joint
  - Given:

$$\ddot{\vartheta}_{\max} \Rightarrow \tau = J_{eq}\ddot{\vartheta} \Rightarrow \tau_{\max} = J_{eq}\ddot{\vartheta}_{\max} = J_{load}TR^2\ddot{\vartheta}_{\max}$$

- Then taking into account some more realistic components:

$$\tau_{\max} = J_{load}\frac{TR^2}{\eta}\ddot{\vartheta}_{\max}$$

Example (continued)

$$\tau_{\max} = J_{load}\frac{TR^2}{\eta}\ddot{\vartheta}_{\max}$$

$$P = \tau_{\max}\dot{\vartheta} \Rightarrow \text{given } \dot{\vartheta}_{\max} \Rightarrow \text{get } P$$

motor power, from catalog

More on real world components

- Efficiency
- Eccentricity
- Backlash
- Vibrations

- To get better results during design mechanical systems can be simulated