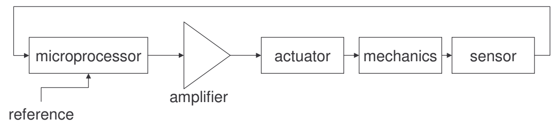
Robotica Antropomorfa

Lezione 4

New version

Control of a single joint



Components

- Digital microprocessor (DSP)
- Amplifier (drives the motor)
- Actuator
- Mechanics/load
- Sensors

Analysis tools

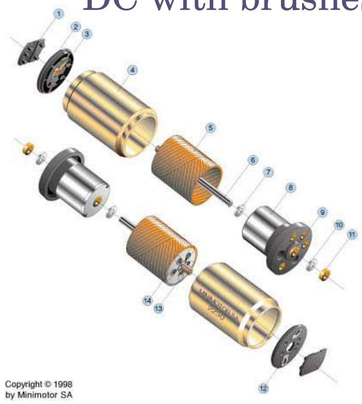
- Root locus
- Frequency response

Actuators

- Various types:
  - AC, DC, etc.
  - DC
    - Brushless
    - With brushes
- We'll have a look at the DC with brushes, simple to control, widely used in robotics

DC-brushless

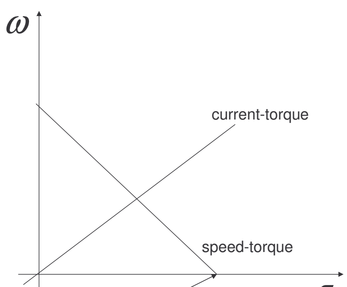
DC with brushes



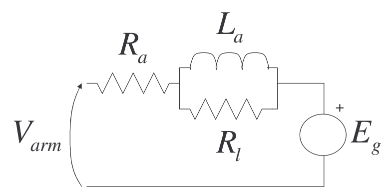
Modeling the DC motor

- High stall torque
- Speed-torque and torque-current relationships are linear

In particular



Electrical schema



$$E_g = \omega(t) K_E$$

Meaning of components

- R_a — Armature resistance (including brushes)
- V_{arm} — Armature voltage
- R_l — Losses due to magnetic field
- E_g — Back EMF produced by the rotation of the armature in the field
- L_a — Coil inductance

We can write…

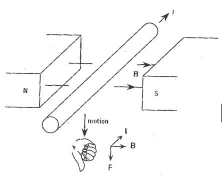
$$V_{arm} = R_a I_a + L_a \dot{I}_a + \omega(t) K_E$$

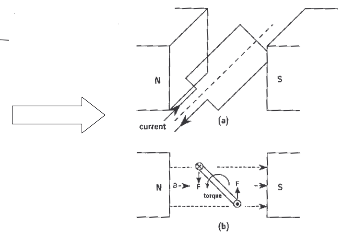
$$\text{for } R_l \ll R_a$$

which is the case at the frequency of interest, and we also have…

$$\tau = K_T I_a$$

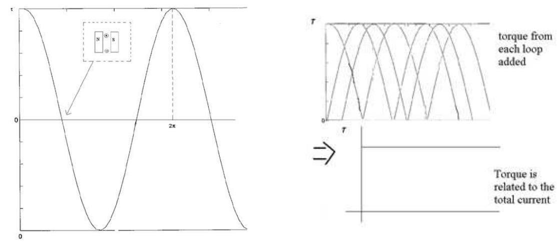
On torque and current





$$F = I \times B$$

Thus for many coils…



Back to motor modeling…

$$\tau = (J_M + J_L)\dot{\omega}(t) + B\omega(t) + \tau_f + \tau_{gr}$$

- τ • Torque generated
- J_M • Inertia of the motor
- J_L • Inertia of the load
- τ_f • Friction
- τ_{gr} • Gravity

Furthermore…

$$V_{arm} = R_a I_a + L_a \dot{I}_a + \omega(t) K_E$$

$$\tau = K_T I_a$$

$$\tau = (J_M + J_L)\dot{\omega}(t) + B\omega(t) + \tau_f + \tau_{gr}$$

By Laplace-transforming

$$V_{arm}(s) = R_a I_a(s) + L_a I_a(s)s + \omega(s)K_E \Rightarrow I_a(s) = \frac{V_{arm}(s) - \omega(s)K_E}{R_a + L_a s}$$

$$\tau = K_T I_a$$

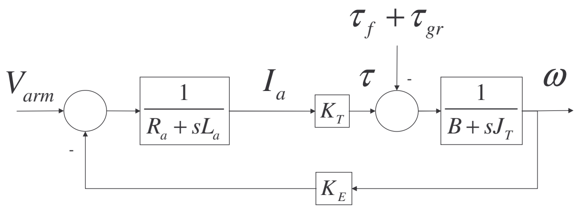
$$K_T \frac{V_{arm}(s) - \omega(s)K_E}{R_a + L_a s} = (J_M + J_L)\omega(s)s + B\omega(s) + \tau_f + \tau_{gr}$$

and finally

$$\frac{\omega(s)}{V_{arm}(s)} = \frac{K_T / L_a J_T}{s^2 + [(R_a J_T + L_a B)/L_a J_T]s + (K_T K_E + R_a B)/L_a J_T}$$

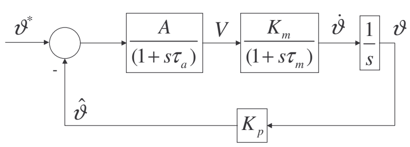
- Considering gravity and friction as additional inputs

Block diagram



A simpler model

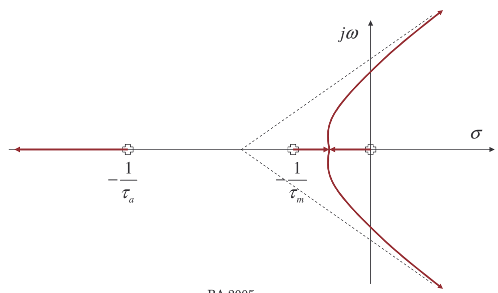
First block diagram



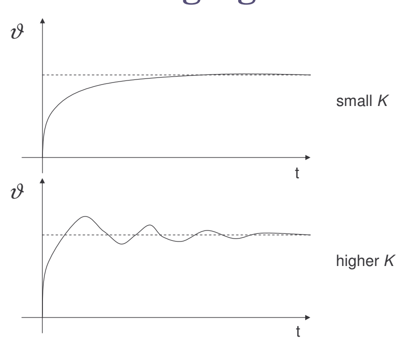
$$H_{open\_loop} = \frac{A}{1+s\tau_a}\frac{K_m}{1+s\tau_m}\frac{K_p}{s}$$

Root locus

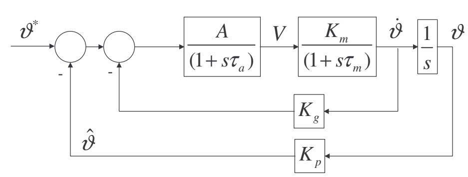
$$H_{open\_loop} = \frac{A}{1+s\tau_a}\frac{K_m}{1+s\tau_m}\frac{K_p}{s} \qquad K = AK_mK_p$$



Changing *K*



Let's add something second diagram



$$H_{open\_loop} = \frac{AK_m(K_p + sK_g)}{(1+s\tau_a)(1+s\tau_m)s}$$

Analysis

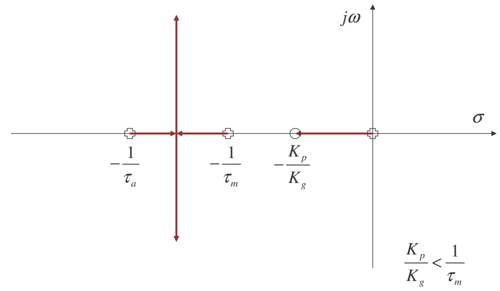
$$H_{open\_loop} = \frac{AK_mK_p(1+s\,{}^{K_g}\!/\!{}_{K_p})}{(1+s\tau_a)(1+s\tau_m)s}$$

$$K = AK_mK_p$$

$$Z_{feedback} = \frac{K_g}{K_p}$$

RA 2005

Root locus (case 1)



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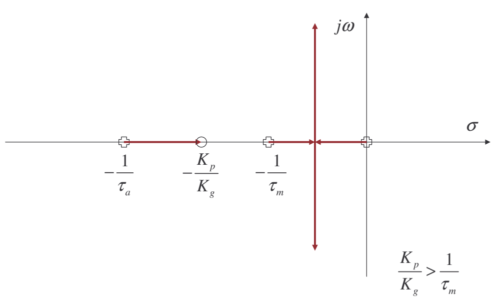
Root locus (case 2)



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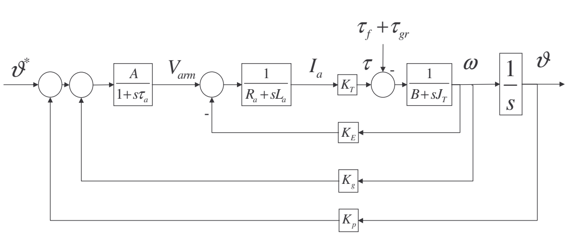
Full model

OS 2005

Back one lesson



RA 2005

Error and performance

$$\vartheta = \frac{\vartheta_d}{s} \qquad M(s) = \frac{K_T}{(R_a+sL_a)(B+sJ_T)+K_EK_T}$$

closed loop (velocity)

$$\vartheta(s) = \frac{1}{s}\omega(s)$$

$$\omega(s) = \frac{\frac{A}{1+s\tau_a}M(s)}{1+\frac{A}{1+s\tau_a}M(s)K_g}$$

closed loop (position)

$$\vartheta(s) = \frac{\frac{1}{s}\omega(s)}{1+\frac{1}{s}\omega(s)K_p}$$

RA 2005

finally

$$\lim_{s\to 0} sH(s) = \lim_{t\to\infty} h(t)$$

$$\Rightarrow \lim_{s\to 0} s\frac{\vartheta_d}{s}\vartheta(s) = \lim_{s\to 0}\frac{s\frac{1}{s}\frac{\vartheta_d}{s}\omega(s)}{1+\frac{1}{s}\omega(s)K_p} = \frac{\vartheta_d}{K_p}$$

- For zero error K must be 1 or the control structure must be different

Same line of reasoning

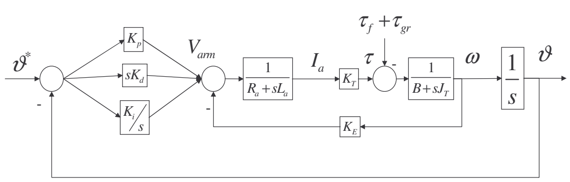
$$\vartheta_{final} = -\frac{T_L R_a}{AK_T K_p}$$

- Final value due to friction and gravity

$$\left|\frac{T_L R_a}{AK_T K_p}\right| \le \vartheta_{\max} \Rightarrow K_p \ge \frac{T_L R_a}{AK_T \vartheta_{\max}}$$

$$K_{p\min} = \frac{T_L R_a}{AK_T \vartheta_{\max}}$$

PID controller



PID controller

- We now know why we need the proportional
- We also know why we need the derivative
- Finally, we add the integral
  - Integrates the error, in practice needs to be limited

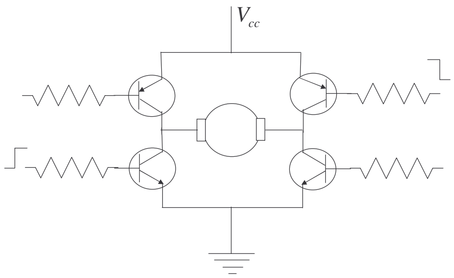
Interpreting the PID

- Proportional: to go where required, linked to the steady-state error
- Derivative: damping
- Integral: to reduce the steady-state error

About the amplifiers

- Linear amplifiers
  - H type
  - T type
- PWM (switching) amplifiers

Let's consider the linear as a starting point



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H-type

- The motor doesn't have a reference to ground (floating)
- It's difficult to get feedback signals (e.g. to measure the current flowing through the motor)

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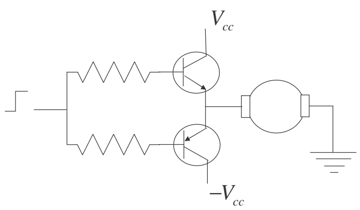
T-type



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On the T-type

- Bipolar DC supply
- Dead band (around zero)
- Need to avoid simultaneous conduction (short circuit)

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Things not shown

- Transistor protection (currents flowing back from the motor)
- Power dissipation and heat sink
  - Cooling
- Sudden stop due to obstacles
  - High currents → current limits and timeouts

RA 2005