Robotica Antropomorfa

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Open Questions

•How can we predict the trajectory followed by the system when it is driven by a given force field *F*? (Dynamic model of the limb)

•Is there a way of choosing the 'complete' set of elementary force fields *Fk*? (A trivial solution to the spinal field synthesis problem)

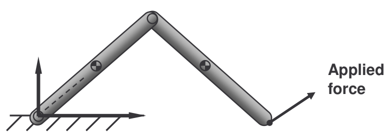
•How should we choose joint torques τ so as to obtain a given basis force field *Fk*? (The map τ -> *Fk*)

•How do we choose muscle activations so as to obtain a given joint torque?

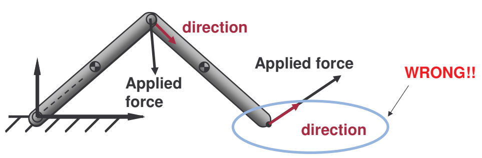
•Which is the minimum number of elementary force fields that we need to perform a 'complete' set of movements?

•Is there a way of choosing the primitives to accommodate different kinematic structures?

A (not so trivial) observation



In which direction will the end-effector move?



Control Model of the spinal field experiment

The spinal fields experiment has been modeled in terms of the linear superposition of a finite number of force fields:

$$\vec{F}(P, v_P) = \sum_{k=1}^{K} \lambda_k \vec{F}_k(P, v_P)$$

Basis field should be convergent to an equilibrium

Force fields in this model depend also on the velocity of *P*. This new feature is justified if we want to introduce a certain degree of damping in the system.

Today we use a different notation:

End effector position: $P \leftrightarrow z$

End effector velocity: $v_P \leftrightarrow \dot{z}$

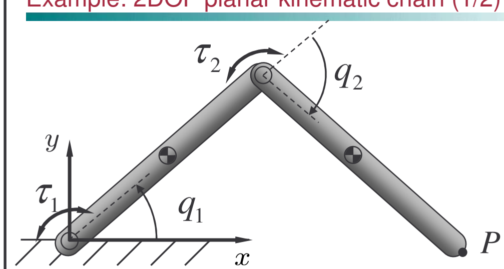
$$\vec{F}(z, \dot{z}) = \sum_{k=1}^{K} \lambda_k \vec{F}_k(z, \dot{z})$$

Modelling a limb as a kinematic chain

A kinematic chained has the following properties:

- It is composed by *n* links $L_1, \ldots, L_n$
- L_j is attached to L_{j-1} by a 1 DOF rotational joint (non restrictive assumption)
- the joint angle (between L_{j-1} and L_j) is denoted q_j
- the end-effector position will be denoted *z* and belongs to an m-dimensional space, with m≤n.

Example: 2DOF planar kinematic chain (1/2)



$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$ **Vector of joint angles**

$$z = \begin{bmatrix} x_P \\ y_P \end{bmatrix}$$ **Vector of end-effector position**

Example: 2DOF planar kinematic chain(2/2)

- Direct kinematics:

First link length

Second link length

$$z=\begin{bmatrix}x_P\\ y_P\end{bmatrix}=\begin{bmatrix}l_1c_{q1}+l_2c_{q1+q2}\\ l_1s_{q1}+l_2s_{q1+q2}\end{bmatrix}=\Lambda(q)$$

Sine of q1

Cosine of q1

- Jacobian:

$$\dot z=\begin{bmatrix}\dot x_P\\ \dot y_P\end{bmatrix}=\begin{bmatrix}-l_1s_{q1}-l_2s_{q1+q2} & -l_2s_{q1+q2}\\ -l_1c_{q1}-l_2c_{q1+q2} & -l_2c_{q1+q2}\end{bmatrix}\begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix}=J(q)\dot q$$

Dynamic model of the limb (1/3) “repetita iuvant”

- The dynamic model of a kinematic chain describes the map from applied forces to trajectories of the joint variables. Let the applied forces be the vector of applied torques. Then:

$$\underbrace{\begin{bmatrix}\tau_1(t)\\ \vdots\\ \tau_n(t)\end{bmatrix} \quad t\in[t_1 \quad t_2]}_{\text{Time evolution of the applied torques}} \xrightarrow{\text{Integration of the dynamic model}} \underbrace{\begin{bmatrix}q_1(t)\\ \vdots\\ q_n(t)\end{bmatrix} \quad t\in[t_1 \quad t_2]}_{\text{Time evolution of the joint angles}}$$

Dynamic model of the limb (2/3) “repetita iuvant”

- The dynamic model can be computed following the Lagrangian approach:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q}-\frac{\partial L}{\partial q}=\tau \qquad \text{where} \qquad L(q,\dot q)=\underbrace{K(q,\dot q)}_{\text{Kinetic energy}}-\underbrace{V(q)}_{\text{Potential energy}}$$

computations

$$M(q)\ddot q+C(q,\dot q)\dot q+G(q)=\tau$$

Inertia Matrix

Coriolis matrix

Gravity effect

Dynamic model of the limb (3/3)

- LEMMA: the matrices of the dynamic model of a kinematic chain satisfy the following two properties:

$$(1) \qquad M(q)=M(q)^T>0$$

Follows from the fact that the kinetic energy is ≥ zero and equals zero only at rest

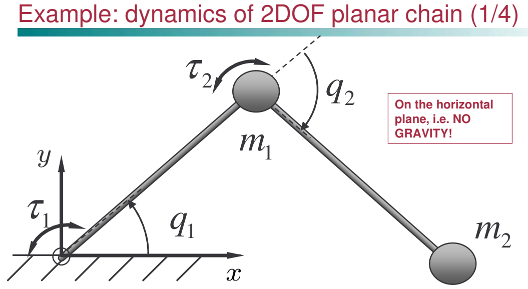
$$(2) \qquad \dot M(q)-2C(q,\dot q) \quad \text{is skew symmetric}$$

(passivity property) It implies that in absence of friction the total energy of the system is conserved

A given matrix A is skew symmetric if and only if:

$$A=-A^T$$

Example: dynamics of 2DOF planar chain (1/4)



On the horizontal plane, i.e. NO GRAVITY!

$$K(q,\dot q)=\frac{1}{2}m_1\|v_{m1}(q,\dot q)\|^2+\frac{1}{2}m_2\|v_{m2}(q,\dot q)\|^2$$

Velocity of m1

Velocity of m2

Example: dynamics of 2DOF planar chain (2/4)

- Computing the velocities:

$$\begin{bmatrix}x_{m1}\\ y_{m1}\end{bmatrix}=\begin{bmatrix}l_1c_{q1}\\ l_1s_{q1}\end{bmatrix} \qquad v_1(q,\dot q)=\begin{bmatrix}\dot x_{m1}\\ \dot y_{m1}\end{bmatrix}=\begin{bmatrix}-l_1s_{q1} & 0\\ l_1c_{q1} & 0\end{bmatrix}\begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix}$$

$$\begin{bmatrix}x_{m2}\\ y_{m2}\end{bmatrix}=\begin{bmatrix}l_1c_{q1}+l_2c_{q1+q2}\\ l_1s_{q1}+l_2s_{q1+q2}\end{bmatrix} \qquad v_2(q,\dot q)=\begin{bmatrix}\dot x_{m2}\\ \dot y_{m2}\end{bmatrix}=\begin{bmatrix}-l_1s_{q1}-l_2s_{q1+q2} & -l_2s_{q1+q2}\\ l_1c_{q1}+l_2c_{q1+q2} & l_2c_{q1+q2}\end{bmatrix}\begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix}$$

$$\|v_1(q,\dot q)\|^2=[\dot q_1 \quad \dot q_2]\begin{bmatrix}l_1^2 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix} \qquad \|v_2(q,\dot q)\|^2=[\dot q_1 \quad \dot q_2]\begin{bmatrix}l_1^2+l_2^2+2l_1l_2c_{q2} & l_2^2+l_1l_2c_{q2}\\ l_2^2+l_1l_2c_{q2} & l_2^2\end{bmatrix}\begin{bmatrix}\dot q_1\\ \dot q_2\end{bmatrix}$$

Example: dynamics of 2DOF planar chain (3/4)

- Kinetic energy:

$$K(q,\dot q)=\frac{1}{2}\dot q^T\begin{bmatrix} \overbrace{m_1 l_1^2+m_2 l_1^2+m_2 l_2^2}^{\alpha}+2\overbrace{m_2 l_1 l_2}^{\beta} c_{q2} & \overbrace{m_2 l_2^2}^{\delta}+m_2 l_1 l_2 c_{q2} \\ m_2 l_2^2+m_2 l_1 l_2 c_{q2} & m_2 l_2^2 \end{bmatrix}\dot q$$

$$=\frac{1}{2}\dot q^T\begin{bmatrix} \alpha+2\beta c_{q2} & \delta+\beta c_{q2} \\ \delta+\beta c_{q2} & \delta \end{bmatrix}\dot q=\frac{1}{2}\dot q^T M(q)\dot q$$

This matrix will turn out to be the inertia matrix

- Dynamics:

No potential energy since we are not considering gravity

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q}-\frac{\partial L}{\partial q}=\tau \quad \textbf{where} \quad L(q,\dot q)=K(q,\dot q)$$

Example: dynamics of 2DOF planar chain (4/4)

$$\frac{\partial L}{\partial \dot q}=\frac{\partial}{\partial \dot q}\left[\frac{1}{2}\dot q^T M(q)\dot q\right]=M(q)\dot q$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q}=\frac{d}{dt}[M(q)\dot q]=M(q)\ddot q+\frac{d}{dt}[M(q)]\dot q=M(q)\ddot q+\begin{bmatrix} -2\beta s_{q2}\dot q_2 & -\beta s_{q2}\dot q_2 \\ -\beta s_{q2}\dot q_2 & 0 \end{bmatrix}\dot q$$

$$\frac{\partial L}{\partial q}=\frac{\partial}{\partial q}\left[\frac{1}{2}\dot q^T M(q)\dot q\right]=\begin{bmatrix} 0 & 0 \\ -\beta s_{q2}\dot q_1 & -\beta s_{q2}\dot q_1 \end{bmatrix}\begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot q}-\frac{\partial L}{\partial q}=\tau \Rightarrow \overbrace{\begin{bmatrix} \alpha+2\beta c_{q2} & \delta+\beta c_{q2} \\ \delta+\beta c_{q2} & \delta \end{bmatrix}}^{M(q)}\ddot q+\overbrace{\begin{bmatrix} -\beta s_{q2}\dot q_2 & -\beta s_{q2}(\dot q_1+\dot q_2) \\ \beta s_{q2}\dot q_1 & 0 \end{bmatrix}}^{C(q,\dot q)}\begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix}=\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

Given the applied torques the joint trajectories can be obtained integrating this dynamical equation!

Try to verify the skew symmetry of the matrix $\dot M(q)-2C(q,\dot q)$

State Space form of the dynamic equation

$$M(q)\ddot q+C(q,\dot q)\dot q+G(q)=\tau \Rightarrow \dot x=f(x)+g(x)u$$

Can be rewritten in the standard state space form

$$\dot x=\begin{bmatrix} q \\ \dot q \end{bmatrix} \qquad u=\tau \qquad g(x)=\begin{bmatrix} 0 \\ M^{-1}(q) \end{bmatrix}$$

$$f(x)=\begin{bmatrix} \dot q \\ -M^{-1}(q)[C(q,\dot q)+G(q)] \end{bmatrix}$$

Therefore all tools of (nonlinear) dynamical systems theory can be used!

PD control of a kinematic chain

Without loss of generality let us assume *G(q) = 0*. If this is not the case let us assume that it has been compensated choosing:

$$\tau=\hat\tau+G(q) \Rightarrow M(q)\ddot q+C(q,\dot q)\dot q=\hat\tau$$

- FACT: the following PD (proportional + derivative) control law:

$$\hat\tau=-K_v\dot q-K_p(q-q_d)$$

Is such that the corresponding system has a unique equilibrium point (q_d) which is globally asymptotically stable.

- PROOF: (sketch) try to use the following Lypunov function:

$$V(q,\dot q)=\frac{1}{2}\dot q^T M(q)\dot q+\frac{1}{2}(q-q_d)^T K_p(q-q_d)$$

and take advantage of the passivity property.

Back to Bizzi's experiment

In case of non-redundant manipulators, we the following equivalences:

$$\vec F \leftrightarrow \tau \qquad z \leftrightarrow q \qquad \dot z \leftrightarrow \dot q$$

And therefore the spinal field model can be rewritten as follows:

$$\vec F(z,\dot z)=\sum_{k=1}^{K}\lambda_k \vec F_k(z,\dot z) \Rightarrow \tau(q,\dot q)=\sum_{k=1}^{K}\lambda_k \tau_k(q,\dot q)$$

Basis field should be convergent to an equilibrium

PRB: How should we choose the elementary control actions so that:

1. **Each elementary controller should drive the system towards a unique (globally asymptotically stable) equilibrium point**
2. **The combinations of the elementary controllers should be capable of driving the system to any desired configuration**

A trivial solution to the synthesis problem (1/2)

HINT:

$$\tau_k=-K_v\dot q-K_p(q-q_{d,k})$$

convergent to the equilibrium $q_{d,k}$

And impose the following for all admissible q_d:

$$\sum_{k=1}^{K}\lambda_k\tau_k(q,\dot q)=-K_v\dot q-K_p(q-q_d)$$

Which can be rewritten:

$$\sum_{k=1}^{K}\lambda_k\left[K_v\dot q+K_p(q-q_{d,k})\right]=K_v\dot q+K_p(q-q_d)$$

Which is verified if:

$$\sum_{k=1}^{K}\lambda_k=1 \quad \textbf{and} \quad \sum_{k=1}^{K}\lambda_k q_{d,k}=q_d$$

A trivial solution to the synthesis problem (2/2)

Rewriting the previous expression:

$$n+1\left\{\underbrace{\begin{bmatrix} q_{d,1} & \cdots & q_{d,K} \\ 1 & \cdots & 1 \end{bmatrix}}_{K}\right.\begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_K \end{bmatrix} = \begin{bmatrix} q_d \\ 1 \end{bmatrix}$$

Which has a solution for any q_d if and only if the matrix on the left has full row rank. This observation gives a criteria to choose the equilibrium point realized by the elementary controls.

Moreover, we can have full row rank only if:

$$K \geq n+1$$

This can be proven to be the minimum number of primitives necessary to control a n-DOF kinematic chain!

Back to the end-effector space

In the redundant case we can go back :

$$\tau(q,\dot{q}) = \sum_{k=1}^{K} \lambda_k \tau_k(q,\dot{q}) \quad \Longrightarrow \quad \vec{F}(z,\dot{z}) = \sum_{k=1}^{K} \lambda_k \vec{F}_k(z,\dot{z})$$

Using the following equations:

The direct kinematics can be (locally) inverted:
INVERSE KINEMATIC

$$z = \Lambda(q) \quad \Longrightarrow \quad q = \Lambda_{inv}(z)$$

$$\dot{z} = J(q)\dot{q} \quad \Longrightarrow \quad \dot{q} = J(q)^{-1}\dot{z} \quad \Longrightarrow \quad \dot{q} = [J(\Lambda_{inv}(z))]^{-1}\dot{z}$$

$$\tau(q,\dot{q}) = J^T(q)\vec{F} \quad \Longrightarrow \quad \vec{F}(q,\dot{q}) = J^{-T}(q)\tau(q,\dot{q}) \quad \Longrightarrow \quad \ldots$$

$$\ldots \quad \Longrightarrow \quad \vec{F}(z,\dot{z}) = J^{-T}(\Lambda_{inv}(z))\tau(\Lambda_{inv}(z), J(q)\dot{z})$$

Graphical representation of the fields(2DOF chain)

$$\tau_k(q,\dot{q}) \qquad\qquad \vec{F}_k(z,\dot{z})$$

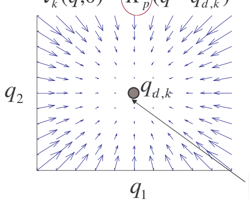
Can be graphically represented (null velocities): $\tau_k(q,0)$

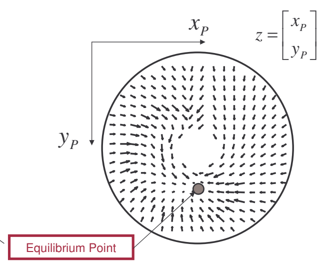
Looks quite different in the Cartesian space: $F_k(z,0)$

EXAMPLE

Identity

$$\tau_k(q,0) = K_p(q - q_{d,k})$$





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