Robotica Antropomorfa

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Open Questions

- How can we predict the trajectory followed by the system when it is driven by a given force field *F*? (Dynamic model of the limb)
- Is there a way of choosing the 'complete' set of elementary force fields *Fk*? (A trivial solution to the spinal field synthesis problem)
- How should we choose joint torques τ so as to obtain a given basis force field *Fk*? (The map τ -> *Fk*)
- How do we choose muscle activations so as to obtain a given joint torque?
- Which is the minimum number of elementary force fields that we need to perform a 'complete' set of movements?
- Is there a way of choosing the primitives to accommodate different kinematic structures?

A trivial solution to the synthesis problem (1/2)

HINT:

The gain matrix do not change with k

$$\tau_k = -K_v\dot{q} - K_p\left(q - q_{d,k}\right)$$

convergent to the equilibrium $q_{d,k}$

And impose the following for all admissible q_d:

$$\sum_{k=1}^{K}\lambda_k\tau_k(q,\dot{q}) = -K_v\dot{q} - K_p\left(q - q_d\right)$$

Which can be rewritten:

$$\sum_{k=1}^{K}\lambda_k\left[K_v\dot{q} + K_p\left(q - q_{d,k}\right)\right] = K_v\dot{q} + K_p\left(q - q_d\right)$$

Which is verified if:

$$\sum_{k=1}^{K}\lambda_k = 1 \qquad \text{and} \qquad \sum_{k=1}^{K}\lambda_k q_{d,k} = q_d$$

A trivial solution to the synthesis problem (2/2)

Rewriting the previous expression:

$$n+1\left\{\underbrace{\begin{bmatrix} q_{d,1} & \cdots & q_{d,K} \\ 1 & \cdots & 1 \end{bmatrix}}_{K}\right.\begin{bmatrix}\lambda_1 \\ \vdots \\ \lambda_K\end{bmatrix} = \begin{bmatrix} q_d \\ 1\end{bmatrix}$$

Which has a solution for any q_d if and only if the matrix on the left has full row rank. This observation gives a criteria to choose the equilibrium points realized by the elementary controls.

Moreover, we can have full row rank only if:

$$K \geq n+1$$

This can be proven to be the minimum number of primitives necessary to control a n-DOF kinematic chain!

Back to the end-effector space

In the redundant case we can go back and forth:

$$\tau(q,\dot{q}) = \sum_{k=1}^{K}\lambda_k\tau_k(q,\dot{q}) \iff \vec{F}(z,\dot{z}) = \sum_{k=1}^{K}\lambda_k\vec{F}_k(z,\dot{z})$$

Using the following equations:

The direct kinematics can be (locally) inverted: INVERSE KINEMATIC

$$z = \Lambda(q) \implies q = \Lambda_{inv}(z)$$

$$\dot{z} = J(q)\dot{q} \implies \dot{q} = J(q)^{-1}\dot{z} \implies \dot{q} = \left[J(\Lambda_{inv}(z))\right]^{-1}\dot{z}$$

$$\tau(q,\dot{q}) = J^T(q)\vec{F} \implies \vec{F}(q,\dot{q}) = J^{-T}(q)\tau(q,\dot{q}) \implies \ldots$$

$$\ldots \implies \vec{F}(z,\dot{z}) = J^{-T}(\Lambda_{inv}(z))\tau(\Lambda_{inv}(z), J(q)\dot{z})$$

Graphical representation of the fields(2DOF chain)

$$\tau_k(q,\dot{q}) \qquad\qquad \vec{F}_k(z,\dot{z})$$

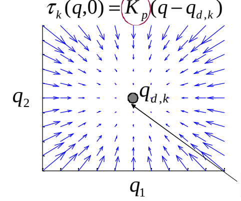
Can be graphically represented (null velocities): $\tau_k(q,0)$

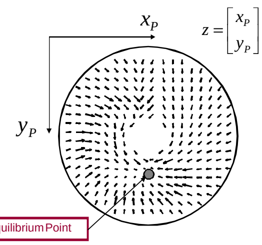
Looks quite different in the Cartesian space: $\vec{F}_k(z,0)$

EXAMPLE

Identity

$$\tau_k(q,0) = K_p\left(q - q_{d,k}\right)$$

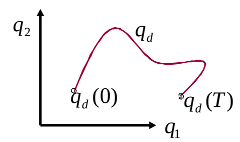




Equilibrium Point

Trajectory tracking

We have seen how to choose the control action so as to drive the system to a predefined (global) equilibrium. Sometimes we are interested in tracking a given trajectory.



Specifically, we are interested in finding a control action such that a given trajectory is asymptotically tracked i.e. the tracking error:

$$e = q - q_d$$

asymptotically tends to zero.

COMPUTED TORQUE control of a chain (1/2)

Suppose that you know the matrices M, C, G of the dynamical model:

$$\tau = M(q)(\ddot{q}_d + \hat{\tau}) + C(q,\dot{q})\dot{q} + G(q) \qquad \text{Control Action}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad \text{System Dynamics}$$

$$\Downarrow$$

$$M(q)\ddot{q} = M(q)(\ddot{q}_d + \hat{\tau})$$

NOTE: M(q) is invertible

$$\Downarrow$$

$$\ddot{q} = \ddot{q}_d + \hat{\tau}$$

$$\hat{\tau} = -K_v\dot{e} - K_p e$$

$$\Downarrow$$

$$\ddot{e} = -K_v\dot{e} - K_p e$$

Tracking error $e = q - q_d$

We now prove that: $e \xrightarrow{t\to\infty} 0$

COMPUTED TORQUE control of a chain (2/2)

- FACT: If the matrices K_p and K_v are symmetric and positive definite then the 'computed torque' control law results in **exponential trajectory tracking** i.e. $e \xrightarrow{t\to\infty} 0$
- PROOF: First observe that the dynamics of e are linear and can be rewritten as follows:

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}}^{A}\begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

We want to prove that all the eigenvalues of A have negative real part. Let λ be one eigenvalue and (v_1, v_2) be the corresponding eigenvector. Then we have:

$$\lambda\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} v_2 \\ -K_p v_1 - K_v v_2 \end{bmatrix}$$

Without loss of generality choose v_1 with unitary norm so that we have:

$$\lambda^2 = v_1^*\lambda^2 v_1 = v_1^*\lambda v_2 = -\underbrace{v_1^* K_p v_1}_{\alpha} - \lambda\underbrace{v_1^* K_v v_1}_{\beta}$$

where α>0 and β>0 because K_p and K_v are positive definite. We therefore have:

$$\lambda^2 + \beta\lambda + \alpha = 0 \quad \xRightarrow{\text{Descartes's rule}} \quad \mathrm{Re}(\lambda) < 0$$

Decomposition of the control action

The 'computed torque' control action can be *approximated* with the linear sum of a finite number of elementary control actions:

$$\tau = M(q)(\ddot{q}_d - K_v\dot{e} - K_p e) + C(q,\dot{q})\dot{q} + G(q)$$

Can be written as follows:

$$\tau(q,\dot{q}) = \sum_{k=1}^{K}\lambda_k \tau_k(q,\dot{q},t)$$

Elementary control action

With the following definition:

$$\tau_k(q,\dot{q},t) = M(q)(\ddot{q}^k{}_d - K_v\dot{e} - K_p e) + C(q,\dot{q})\dot{q} + G(q)$$

And choosing combinators according to the following rules:

$$\sum_{k=1}^{K}\lambda_k = 1 \qquad \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_K \end{bmatrix} = \lambda = \arg\min_{\lambda}\left\|\sum_{k=1}^{K}\lambda_k q^k{}_d(\cdot) - q_d(\cdot)\right\|$$

Suitably defined metric in the space of trajectories, e.g.: $\|q(\cdot)\|^2 = \int_0^T q^T(t)q(t)dt$

Control action in the task space(1/2)

Consider the following 'non-redundant' manipulator:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \qquad q \in R^n, x \in R^n$$

End-effector position

And let the Jacobian be:

Jacobian

$$x = h(q) \qquad \dot{x} = J(q)\dot{q} \qquad \dot{q} = J^{-1}(q)\dot{x}$$

Computations lead to:

$$\tilde{M}(q)\ddot{x} + \tilde{C}(q,\dot{q})\dot{x} + \tilde{G}(q) = F$$

where:

$$\tilde{M}(q) = J^{-T}(q)M(q)J^{-1}(q)$$

$$\tilde{C}(q,\dot{q}) = J^{-T}(q)\left[C(q)J^{-1}(q) + M(q)\frac{d}{dt}J^{-1}(q)\right]$$

$$\tilde{G}(q) = J^{-T}(q)G(q)$$

$$F = J^{-T}(q)\tau$$

Control action in the task space (2/2)

- LEMMA: the matrices of the dynamic model of a kinematic chain satisfy the same properties that hold for M and C:

$$(1) \qquad \tilde{M}(q) = \tilde{M}(q)^T > 0$$

Follows from the fact that the kinetic energy is ≥ zero and equals zero only at rest

$$(2) \qquad \dot{\tilde{M}}(q) - 2\tilde{C}(q,\dot{q}) \quad \textbf{is skew symmetric}$$

(passivity property) It implies that in absence of friction the total energy of the system is conserved

A given matrix A is skew symmetric if and only if:

$$A = -A^T$$

Control in the task space

PD Control in the task space:

Stabilizing controller $F = \widetilde{G}(q) - K_v \dot{x} - K_p (x - x_d)$

Tracking Control in the task space:

Tracking error $e = x - x_d$

Tracking controller $F = \widetilde{M}(q)(\ddot{x}_d - K_v \dot{e} - K_p e) + \widetilde{C}(q, \dot{q})\dot{q} + \widetilde{G}(q)$