Neighborhood operations

- Generate an output pixel on the basis of the pixel and its neighbors
- Often involve the *convolution* of an image with a filter *kernel* or *mask*

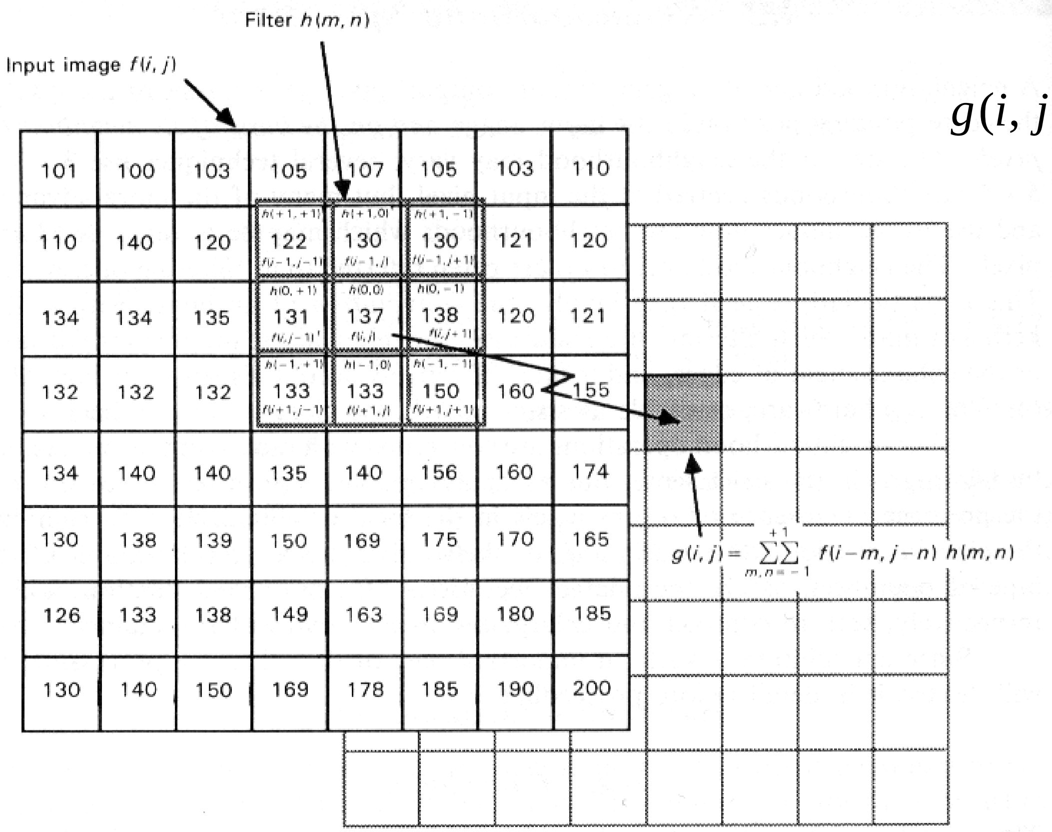


Figure 4.9. Convolution.

$$g(i,j) = f * h = \sum_m \sum_n f(i-m, j-n)h(m,n)$$

```
//example: for a 3x3 kernel
for r=1:H
    for c=1:W
        // for each feasible points in the image
        temp=0.0;
        for m=-1:1
          for n=-1:1
            temp=temp+f(r-m,c-n)*h(m,n);
          end
        end
        g(r,c)=temp;
    end
end
```

computational cost, order of *mxnxWxH* for an image *WxH*

D. Vernon; Machine Vision : Automated Visual Inspection and Robot Vision, Prentice Hall, 1991

Esample: noise suppression

- Assuming (usually correctly) that the noise has a high spatial frequency
- Apply a low-pass spatial filter
- Of course high-frequencies in the image will be degraded after filtering…

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

Figure 4.11 Local average mask.

101 * 1/9	100 * 1/9	103 * 1/9	105	107	105	103	110
110 * 1/9	140 * 1/9	120 * 1/9	122	130	130	121	120
134 * 1/9	134 * 1/9	135 * 1/9	131	137	138	120	121
132	132	132	133	133	150	160	155
134	140	140	135	140	156	160	174
130	138	139	150	169	175	170	165
126	133	138	149	163	169	180	185
130	140	150	169	178	185	190	200

$$
\begin{aligned}
&101*1/9+100*1/9+103*1/9 \\
&+110*1/9+140*1/9+120*1/9 \\
&+134*1/9+134*1/9+135*1/9
\end{aligned}
$$

Gaussian smoothing

- The image is convolved with a Gaussian function

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\left(x^2+y^2\right)/2\sigma^2}$$

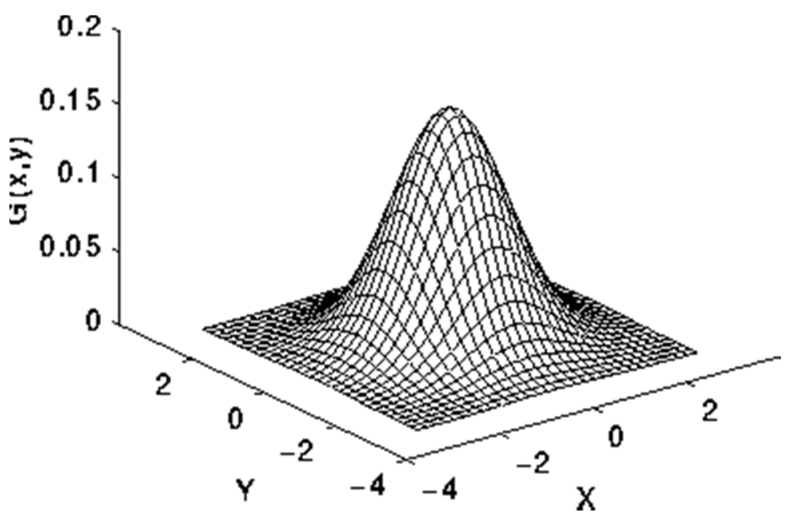
σ defines the spread of the function

small $\sigma \rightarrow$ narrow gaussian

large $\sigma \rightarrow$ broad gaussian

$\frac{1}{273}$

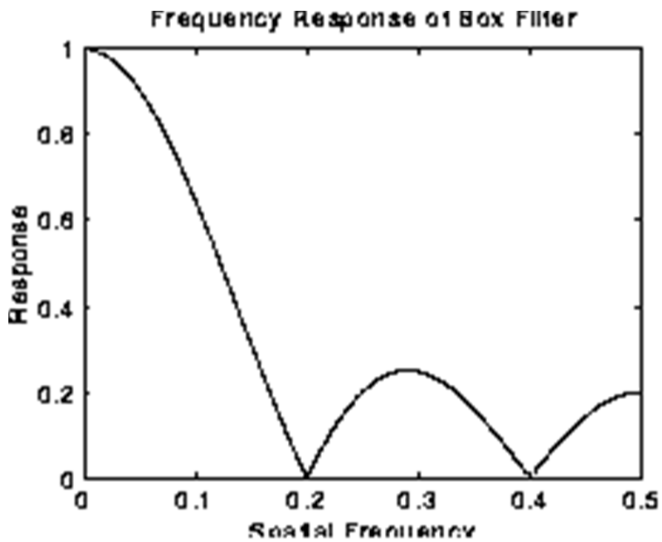
1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

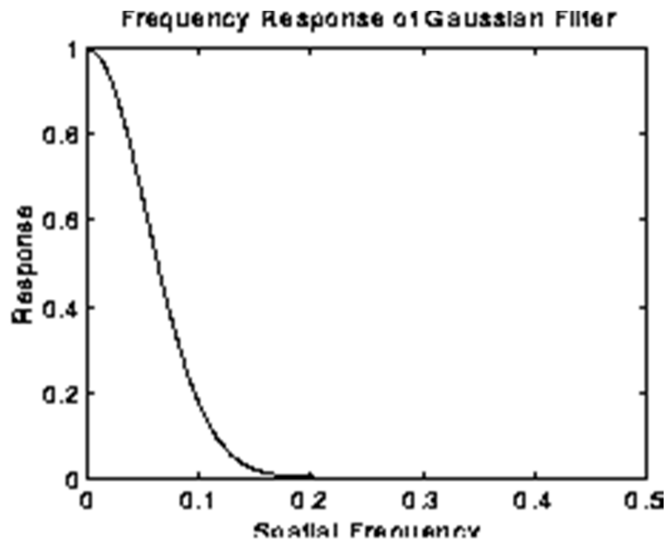


from: http://www.cee.hw.ac.uk/hipr/html/gsmooth.html

Why smooth with a Gaussian? (1)

- The Gaussian filter has nice properties in the frequency domain:





$$\mathcal{F}\left(\mathrm{rect}(ax)\right) = \frac{1}{a} \cdot \mathrm{sinc}\left(\frac{\xi}{a}\right)$$

$$\mathcal{F}\left(e^{-\alpha x^2}\right) = \sqrt{\frac{\pi}{\alpha}} \cdot e^{-\frac{(\pi\xi)^2}{\alpha}}$$

Beware of aliasing…

Why smooth with a Gaussian? (2)

- Convolving a Guassian with a Gaussian, gives a Gaussian:

$$G_{\sigma_1} * G_{\sigma_2} = G_{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

- it is possible to obtain heavily smoothed images by resmoothing smoothed images…

Why smooth with a Gaussian? (3)

- Filtering with a 2D Gaussian can be separated in two convolutions with one dimensional Gaussian function:

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}$$

Why smooth with a Gaussian? (3)

- Filtering with a 2D Gaussian can be separated in two convolutions with one dimensional Gaussian function:

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}$$

$$g(i, j) = f * g = \sum_m \sum_n g(m, n) f(i-m, j-n) =$$

$$= \sum_m \sum_n e^{-\frac{m^2+n^2}{2\sigma^2}} f(i-m, j-n) =$$

$$= \sum_m e^{-\frac{m^2}{2\sigma^2}} \underbrace{\sum_n e^{-\frac{n^2}{2\sigma^2}} f(i-m, j-n)}_{\text{convolve with a vertical 1D Gaussian}} =$$

$$= \underbrace{\sum_m e^{-\frac{m^2}{2\sigma^2}} h'(i-m, j)}_{\text{convolve the result with an horizontal 1D Gaussian}}$$

- Convolving with a separable filter is the same as convolving with two 1D kernels, but faster:

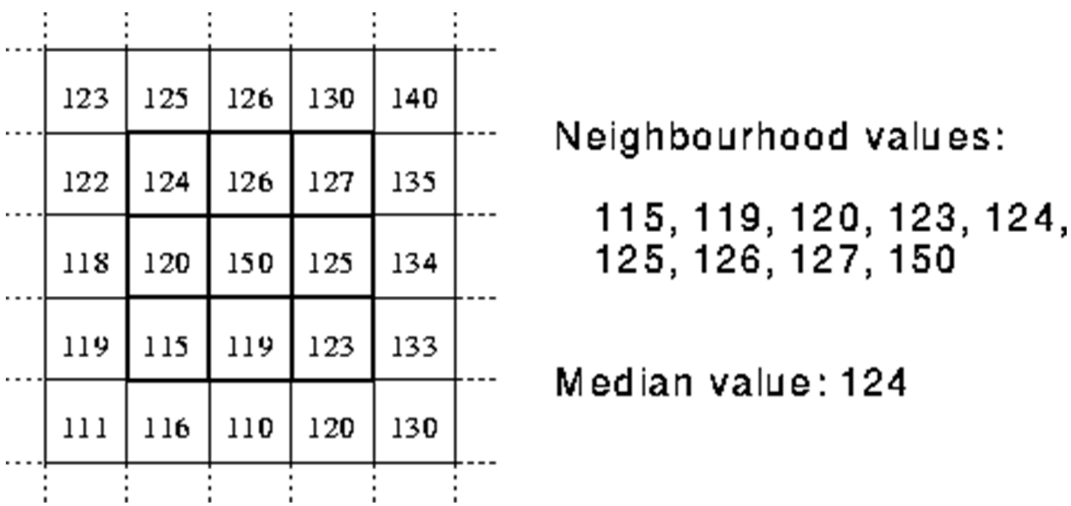
nxHxW + mxHxW operations, instead of nxmxHxW

Example

- Above: images corrupted by normally distributed additive noise (std 5,10,15,20)
- Below: smoothing with a gaussian filter 10x10 std=3

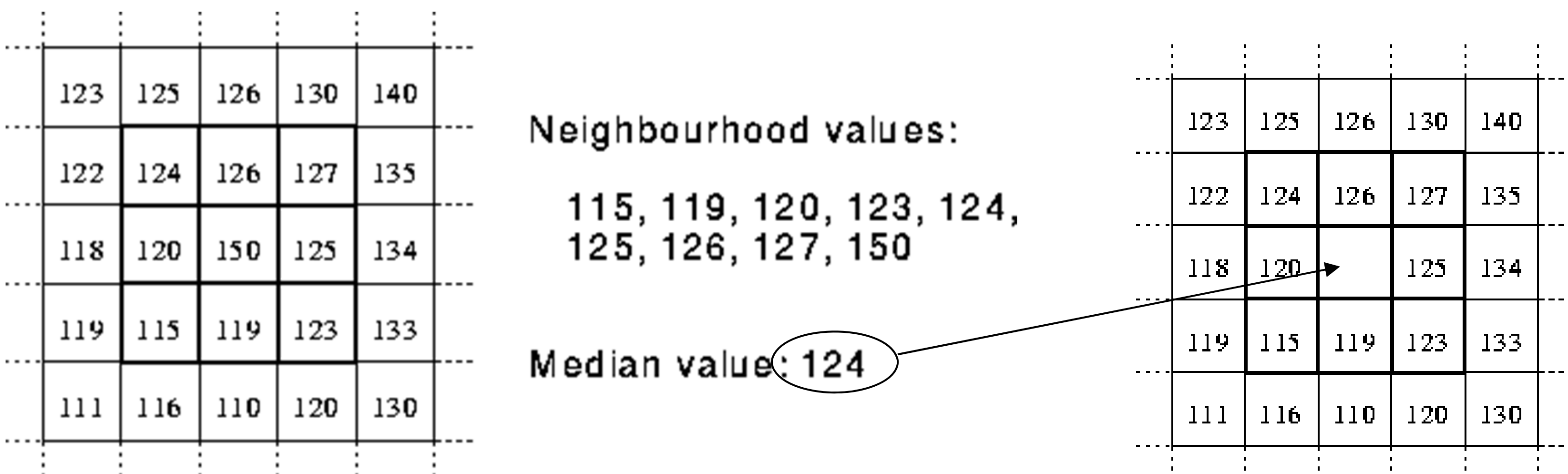
Median Filter

- Non linear technique, useful for noise supression
- In one dimension:
  - slide a window of an odd number of pixel
  - replace the center pixel with the median within the window
- The median of a discrete sequence of N (odd) elements is the number so that (N-1)/2 elements are smaller or equal in value and (N-1)/2 elements are larger or equal in value
  - In practice: sort the pixels and pick the value in the middle…



Median Filter

- Non linear technique, useful for noise supression
- In one dimension:
  - slide a window of an odd number of pixel
  - replace the center pixel with the median within the window
- The median of a discrete sequence of N (odd) elements is the number so that (N-1)/2 elements are smaller or equal in value and (N-1)/2 elements are larger or equal in value
  - In practice: sort the pixels and pick the value in the middle…



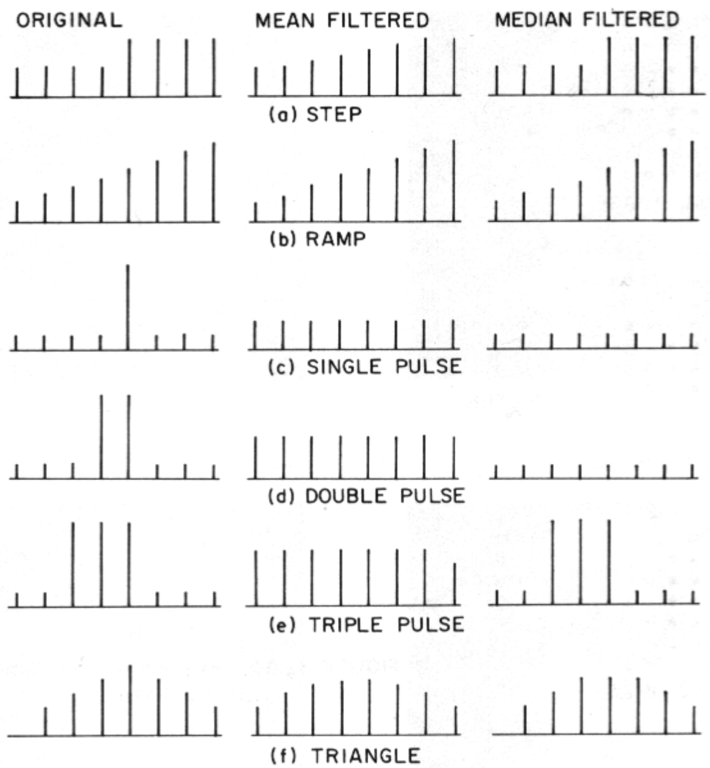


FIGURE 12.6-1. Examples of median filtering on primitive signals, $L = 5$.

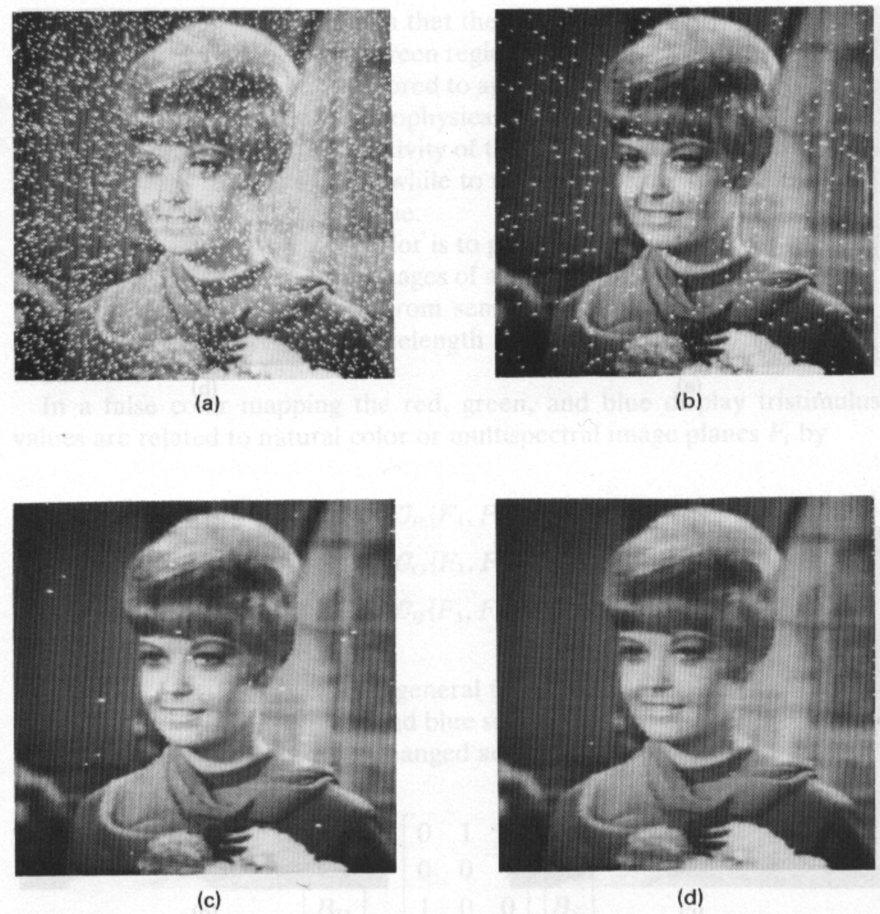


FIGURE 12.6-3. Examples of one-dimensional median filtering for images corrupted by impulse noise. (a) Image with impulse noise, 15 errors per line. (b) Median filtering of (a) with $L = 3$. (c) Median filtering of (a) with $L = 5$. (d) Median filtering of (a) with $L = 7$.

W.K.Pratt, Digital Image Processing

Edge detection

- It is an example of *feature extraction*
- Edges correspond to abrupt changes of luminous intensity in the image
- They usually correspond to discontinuities in the visual scene, due to illumination, object surface or material → object boundaries
- Estimate the gradient of the image:

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

- The *rate of change* of the image is maximum along the direction:

$$\vartheta = \arctan\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x} \right)$$

- With magnitude:

$$\left|\nabla f\right| = g(x,y) = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

- Simplest way to estimate the derivatives:

$$\frac{\partial f}{\partial x} = f(x+1,y) - f(x,y), \quad \frac{\partial f}{\partial y} = f(x,y+1) - f(x,y)$$

- Another approach (*Roberts*) compute the derivatives diagonally over a 2x2 region:

$$g(x,y) \approx R(x,y) = \sqrt{\left[f(x+1,y+1) - f(x,y)\right]^2 + \left[f(x+1,y) - f(x,y+1)\right]^2}$$

- First differences are sensitive to noise, a better approach is to combine differencing with local averaging. For example *Sobel* operator:

$$S_y = \left[f(x-1,y+1) + 2f(x,y+1) + f(x+1,y+1)\right] - \left[f(x-1,y-1) + 2f(x,y-1) + f(x+1,y-1)\right]$$

Takes the difference of a weighted average of the image intensity of either sides of *f(x,y)*

$$S_x = \left[f(x+1,y-1) + 2f(x+1,y) + f(x+1,y+1)\right] - \left[f(x-1,y-1) + 2f(x-1,y) + f(x-1,y+1)\right]$$

$$g(x,y) \approx \sqrt{S_x^2 + S_y^2}$$

$$or: g(x,y) = |S_x| + |S_y|$$

Δ_1

0	1
−1	0

Δ_2

1	0
0	−1

(a)

−1	0	1
−1	0	1
−1	0	1

1	1	1
0	0	0
−1	−1	−1

(b)

−1	0	1
−2	0	2
−1	0	1

1	2	1
0	0	0
−1	−2	−1

(c)

Edge operators can be represented as convolution kernels:

a) Roberts

b) Prewitt

c) Sobel

$$\frac{\partial I}{\partial x} = K_x * I$$

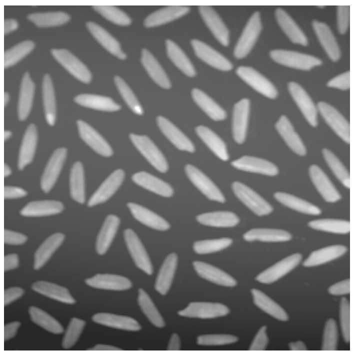
$$\frac{\partial (S * I)}{\partial x} = (K_x * S) * I$$

$$\frac{\partial (S * I)}{\partial x} = \frac{\partial S}{\partial x} * I$$

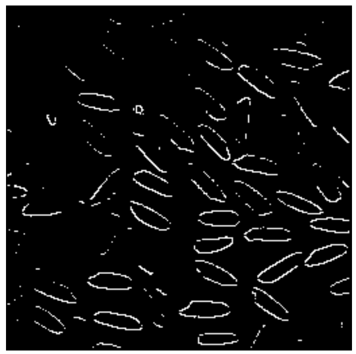
If S is Gaussian then the kernel is a derivative of a G

- Once the gradient magnitude has been estimated, decide if an edge is present or not based on a threshold:

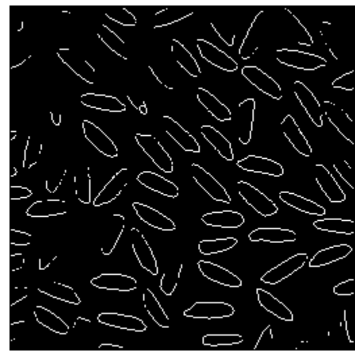
$$edge(x,y) = \begin{cases} 0 & \text{if } g(x,y) < t \\ 1 & \text{otherwise} \end{cases}$$



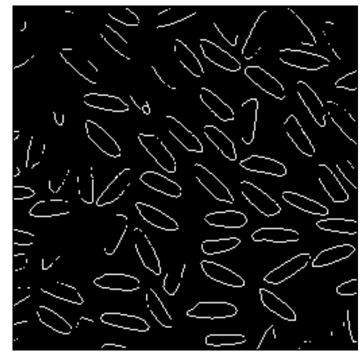
original image (rice.tif)



Roberts (th=0.13)



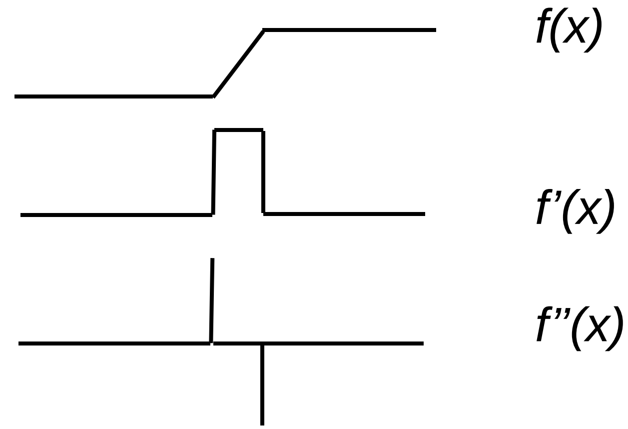
Prewitt (th=0.09)



Sobel (th=0.09)

```
I = imread('rice.tif');
eRob= edge(I, ‘roberts’);
ePre = edge(I,'prewitt');
eSob = edge(I,‘sobel');
figure(1), imshow(eRob)
figure(2), imshow(ePre)
figure(3), imshow(eSob)
```

Laplacian



- Alternative method, computes the Laplacian:

$$\nabla^2 f(x,y) = \left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right)$$

- Approximation:

  1d: f'(x+1)=f(x+1)-f(x), f'(x)=f(x)-f(x-1) → f''(x)=f'(x+1)-f'(x)

  2d: convolve with kernels:

0	-1	0
-1	4	-1
0	-1	0

or

-1	-1	-1
-1	8	-1
-1	-1	-1

- Zero response to linear ramps (gradual changes in intensity), respond to either sides of edges (+/-) → detect edges as zero crossing
- Drawback → strong response to noise
- First convolve with a Gaussian (Marr and Hildreth)

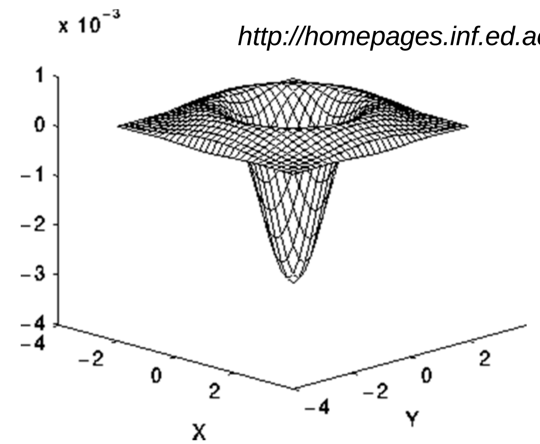
- The Laplacian operator has some nice properties (and is a linear kernel):

$$\nabla^2\left\{G(x,y) * I(x,y)\right\} = \nabla^2 G(x,y) * I(x,y)$$

$\nabla^2 G(x,y)$ is also called LoG (Laplacian of Gaussian)

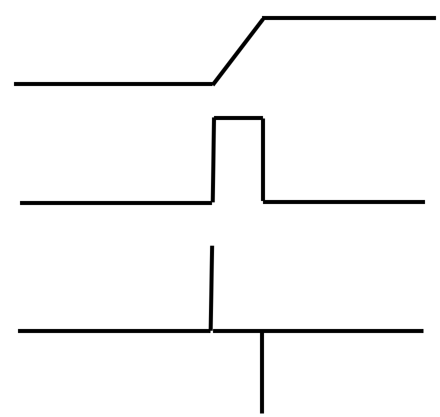
0	1	1	2	2	2	1	1	0
1	2	4	5	5	5	4	2	1
1	4	5	3	0	3	5	4	1
2	5	3	-12	-24	-12	3	5	2
2	5	0	-24	-40	-24	0	5	2
2	5	3	-12	-24	-12	3	5	2
1	4	5	3	0	3	5	4	1
1	2	4	5	5	5	4	2	1
0	1	1	2	2	2	1	1	0

Discrete approximation of a LoG (std=1.4)



http://homepages.inf.ed.ac.uk/rbf/HIPR2/log.htm

- To recap the Laplacian is:
  zero distant from the edge
  > 0 just before the edge
  zero in between the edge
  < 0 after the edge



f(x)

f'(x)

f''(x)

More on the LoG (useful things to know)

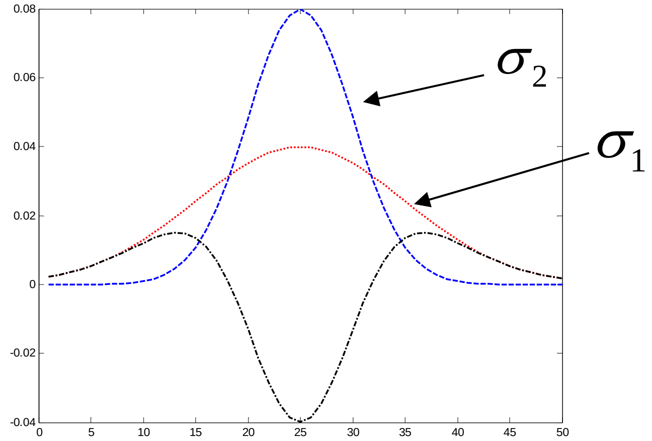
- The two dimensional convolution can be separated into four one-dimensional convolution (cost efficiency):

$$\nabla^2\{I(x,y)*G(x,y)\} = G(x)*\left\{I(x,y)*\frac{\partial^2}{\partial^2 y}G(y)\right\} + G(y)*\left\{I(x,y)*\frac{\partial^2}{\partial^2 x}G(x)\right\}$$

- The LoG can be approximated as (again, easy to compute):

$$g_1(x,y) - g_2(x,y) = G_{\sigma_1} * f(x,y) - G_{\sigma_2} * f(x,y) = \left(G_{\sigma_1} - G_{\sigma_2}\right) * f(x,y) = DoG * f(x,y)$$

$$\sigma_1 > \sigma_2$$



Canny Edge Detector (1986)

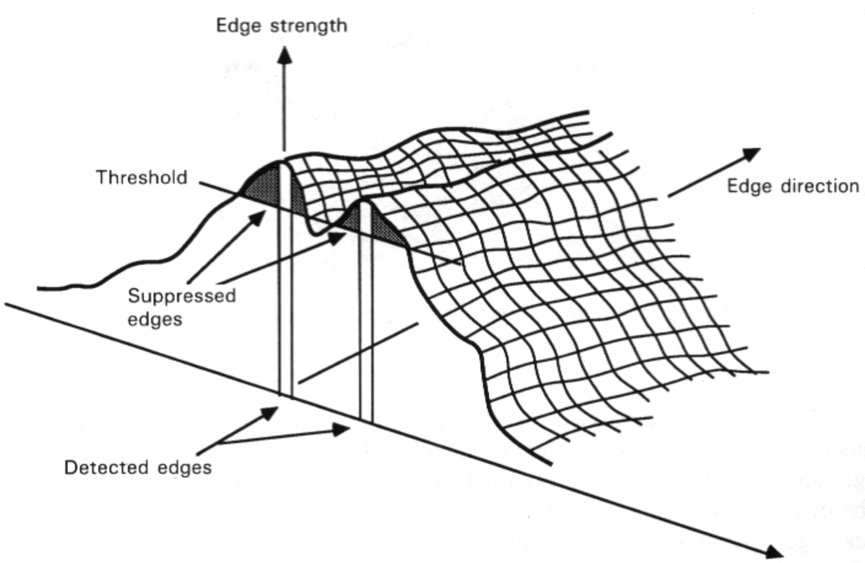
- Problem of the edge detectors: produce thick edges, sometimes edges are not connected because of noise

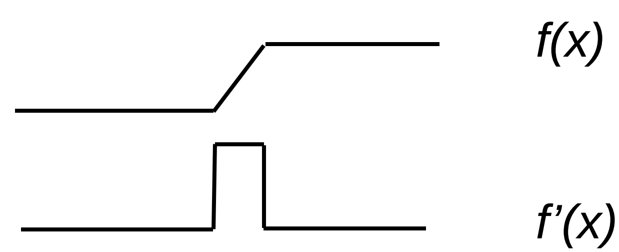
- Smooth with a Gaussian, then apply Sobel
- Thin edges, *non-maxima suppression*:

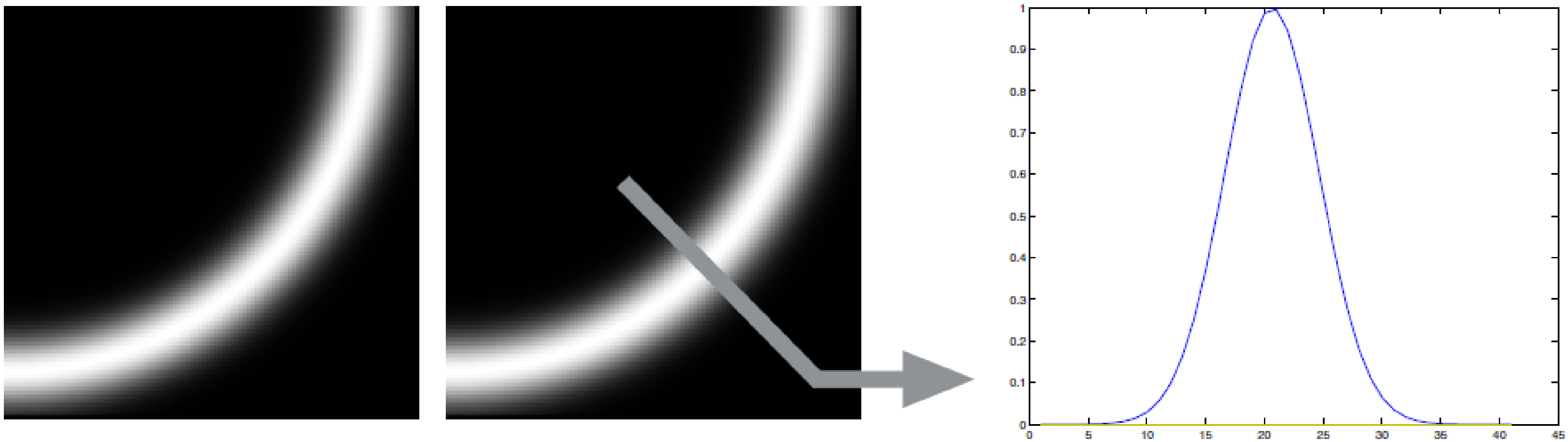
  Edge is found if

  a) response exceeds a given threshold *and*

  b) it is not dominated by responses at neighboring points in a direction normal to the candidate edge → the edge must be higher than the edge magnitude of the pixels on either side



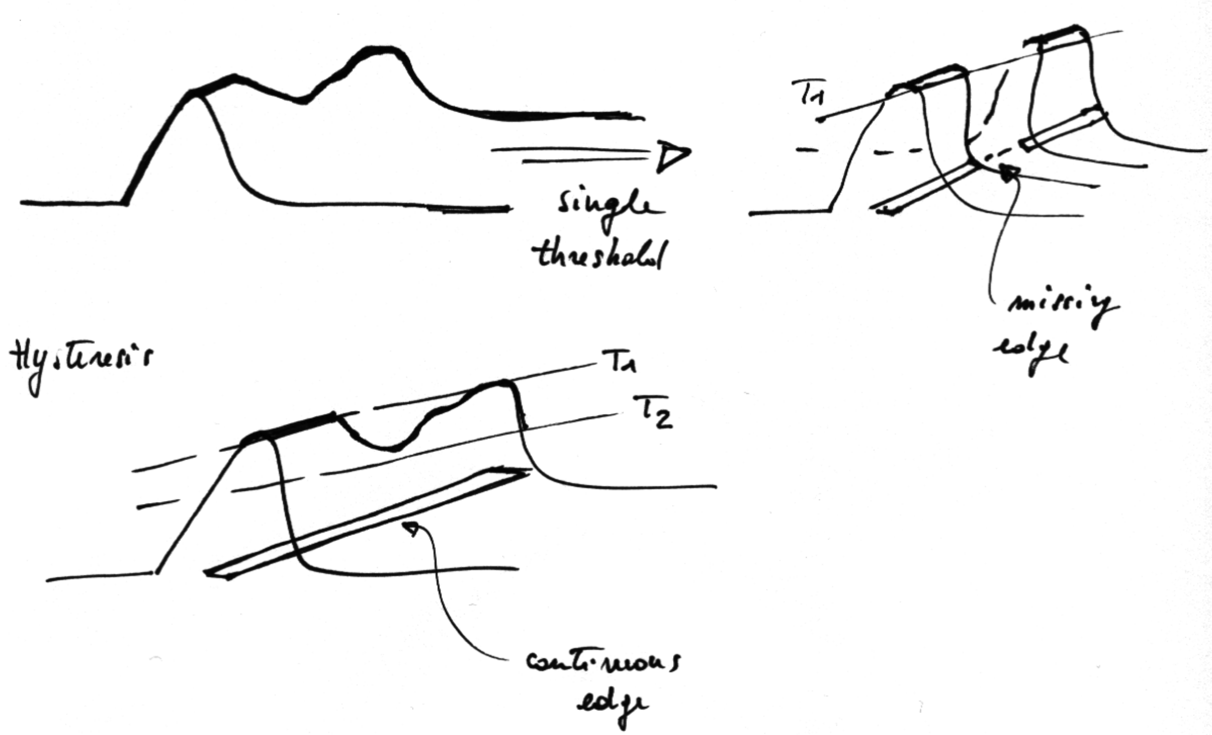




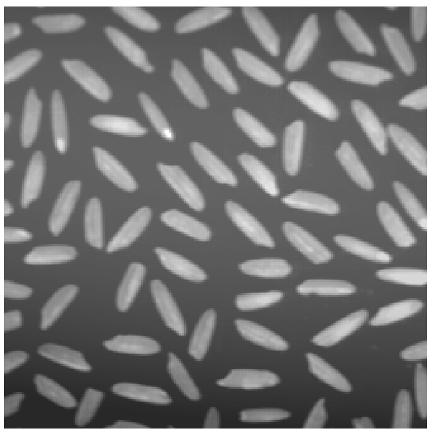
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
30
35
40
45

Canny Edge (2)

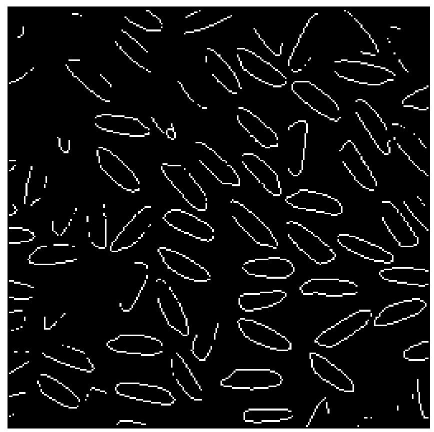
- Extends weak edges: Hysteresis controlled by two thresholds T1 > T2; all pixels above T1 are marked as edge; then all pixels connected to these edges whose value is above T2 will be selected as edge (avoid 'dashed edges')



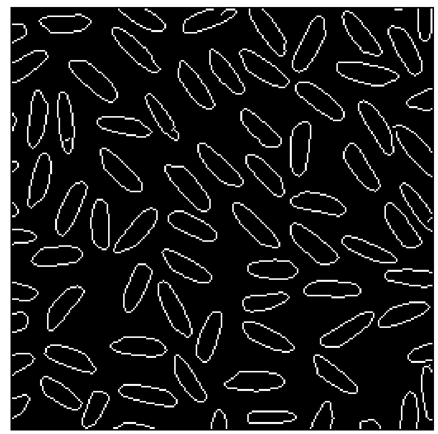
```
While there are points with high gradient
that have not been visited

  Find a start point that is a local maximum in the
    direction perpendicular to the gradient
    erasing points that have been checked

  while possible, expand a chain through
    the current point by:
      1) predicting a set of next points, using
        the direction perpendicular to the gradient

      2) finding which (if any) is a local maximum
        in the gradient direction

      3) testing if the gradient magnitude at the
        maximum is sufficiently large

      4) leaving a record that the point and
        neighbours have been visited

      record the next point, which becomes the current point

  end

end
```



original image (rice.tif)



Sobel (th=0.09)



Canny